Additive Light Field Displays:
Realization of Augmented Reality with Holographic Optical Elements

Seungjae Lee Changwon Jang Seokil Moon Jaebum Cho Byoungho Lee*
Seoul National University

Figure 1: See-through additive light field displays with holographic optical elements (HOEs). We propose a novel compressive light field display that uses HOE layers. Each layer diffuses a projection beam from a projector and functions as an additive layer. This prototype consists of two HOE layers, two projectors, and corresponding relaying optics. The images on the right demonstrate the display results of our prototype, and its high levels of transparency, brightness, and image fidelity.

Abstract

We propose a see-through additive light field display as a novel type of compressive light field display. We utilize holographic optical elements (HOEs) as transparent additive layers. The HOE layers are almost free from diffraction unlike spatial light modulator layers, which makes this additive light field display more advantageous when modifying the number of layers, thickness, and pixel density compared with conventional compressive displays. Meanwhile, the additive light field display maintains advantages of compressive light field displays. The proposed additive light field display shows bright and full-color volumetric images in high definition. In addition, users can view real-world scenes beyond the displays. Hence, we expect that our method can contribute to the realization of augmented reality. Here, we describe implementation of a prototype additive light field display with two additive layers, evaluate the performance of transparent HOE layers, describe several results of display experiments, discuss the diffraction effect of spatial light modulators, and analyze the ability of the additive light field display to express uncorrelated light fields.

Keywords: autostereoscopic 3D displays, holographic optical elements, transparent, light fields, additive layers

Concepts: Computing methodologies → Graphic systems and interface; Computational photography; Hardware → Emerging technologies; Emerging optical and photonic technologies;

1 Introduction

Augmented reality (AR) is a technology that integrates computer-generated information into a live view of a real-world broadcast. AR is regarded as one of the next-generation display technologies since it can be used for various applications [Azuma 1997]. For instance, AR for entertainment or gaming applications can provide fascinating experiences to interact with imaginary objects surrounded by real-world situations. In the medical field, AR can assist doctors in surgery [State et al. 1996] by displaying information of patients such as heartbeat rate, blood pressure, and the location of the tumor. AR can be an effective tool for education [Kaufmann and Schmalstieg 2003] since students can learn math, chemistry, or geometry with visualized structures. Furthermore, AR can support the military with a navigation system or assistance in perceiving the real world.

With the current level of progress in computation speed and optical design skills, the realization of AR is no longer a daydream that is possible only in science fiction movies. However, there remain some issues that should be solved or improved in order to realize more effective and practical AR systems. First, introducing 3-dimensional (3D) information of high-definition to users is a challenging topic. Stereoscopic 3D displays such as high-speed liquid crystal displays (LCDs) with shutter glasses could not successfully attract the public attention since consumers tend to feel antipathy when required to wear equipment for watching displayed images. Autostereoscopic 3D displays suffer from display performances that are determined by viewing angle, image fidelity, and depth of field. Second, implementing transparent displays for AR is another challenging issue. Manufacturing display panels with transparent elements is one solution, but this can be difficult and expensive. A more economical way to realize transparent displays involves the use of beam-combiners in a projection system. Half mirrors, wave guides, and holographic optical elements (HOEs) can be utilized as the beam-combiners. However, additional optical elements make display systems more bulky.

Our aim is to construct a transparent display system that will provide 3D information for AR. With inspiration from previous stud-
ies such as light field displays using multi-layers [Wetzstein et al. 2011; Lanman et al. 2011] and transparent display using HOEs [Oival et al. 2005; Hong et al. 2014], we introduce a novel type of compressive light field display with transparent HOE layers, called the additive light field display. The proposed display utilizes HOE layers as transparent projection screens, which diffuse images from projectors and generate independent 2D light fields. The generated light fields are merged into a single light field via addition, which allows them to form 4D light fields. Therefore, this additive light field display can reconstruct 4D light fields, which can provide a motion-parallax according to the viewing position.

The additive light field display has distinct benefits. First, it is a transparent autostereoscopic 3D display that can be applied to AR. Second, other compressive displays employ a stack of LCDs that involves moiré and results in a loss of brightness, but the additive light field display uses a stack of HOE layers and is relatively free from these problems. Third, the additive light field display is flexible to allow increases in pixel density, while the other compressive displays suffer from diffraction caused by spatial light modulators. In the end, this system has obvious merits for several applications such as head-mounted displays, for which time multiplexing is not essential [Huang et al. 2015].

The contributions of this study are as follows.

- We introduce an additive light field display for use as a transparent 3D display for AR.
- Using HOEs, we implement and evaluate transparent projection screens for the multi-layered displays.
- We describe the design and implementation of a prototype of the additive light field display.
- We analyze the diffraction effect of spatial light modulator layers, evaluate the validity of ray approximation, and suggest a modified projection matrix that includes the diffraction effect.
- We use a statistical approach to analyze the ability of the proposed additive light field display to compress uncorrelated light fields.

2 Related Work

Autostereoscopic 3D Displays Glasses-free 3D displays have received attention from academia and industry since the display systems require no special glasses, which are inconvenient. Several autostereoscopic displays [Lee 2013], such as integral imaging [Lippmann 1908] and holographic displays, have been studied for more than a century. Holographic displays reconstruct complex wavefronts with coherent light sources and spatial light modulators (SLMs). Holographic displays have a wide depth of field and provide depth cues without vergence-accommodation conflict. However, when a spatial bandwidth product (SBP) is settled, holographic displays suffer from diffraction caused by spatial light modulators. In contrast, the proposed additive light field display with HOE layers is relatively free from the diffraction effect since the elements do not have a pixelated structure. Also, additive light field displays are transparent, which is a significant merit among compressive light field displays.

Volumetric Displays with Multi-Focal Planes The additive light field display is closely related to volumetric stereoscopic displays with multi-focal planes. Akeley et al. [2004] and Narain et al. [2015] proposed a stereoscopic volumetric display with a stack of additive layers (beam splitters or LCD panels with time-multiplexing). Switchable lenses are utilized [Liu et al. 2008; Love et al. 2009] to image display panels at a few planes of different depths. The four methods provide focus cues of different depths with additive multi-layered imaging planes. The principle is similar to the additive light field display, which also generates light fields by addition. However, volumetric displays with beam splitters or switchable lenses trade spatial resolution or refresh rates to achieve multi-focal planes, respectively. In particular, the volumetric displays with switchable lenses or LCD panels have limited applications since the viewer position is fixed. On the other hand, additive light field displays are autostereoscopic 3D displays with the potential to be applied to projection-type, head-mounted, or head-up displays.

Holographic Optical Elements D. H. Close [1975] first introduced HOEs, which are volume holograms that transform an incident wavefront to a predefined wavefront when the incident wavefront satisfies the Bragg condition. On the other hand, lights from a real-world scene may pass through HOEs without diffraction since the lights do not satisfy the Bragg condition. This property gives HOEs a distinct characteristic of optical transparency. HOEs have various applications that include polarization-selective optical elements [Huang 1994], see-through lens arrays for integral imaging [Hong et al. 2014; Jang et al. 2016], transparent diffusive screens for projection displays [Yeom et al. 2015], and beam-combiners for head-mounted displays [Kasai et al. 2001]. In fact, transparent diffusive screens and head-mounted displays are ready for commercialization by HoloPro\textsuperscript{1} and DigiLens\textsuperscript{2}.

Augmented Reality AR has recently become one of the most spotlighted technologies based on advances in optical engineering and computer science. For instance, wearable AR displays such

\textsuperscript{1}http://www.holopro.com
\textsuperscript{2}http://www.digilens.com
as Google Glass\(^3\) and HoloLens\(^4\) have attracted considerable public attention. At the Retail Asia Expo 2015, Samsung introduced a transparent organic light-emitting diode (OLED) display that could be utilized as a panel-type device for AR. Hilliges et al. [2012] proposed a HoloDesk that enables users to interact with virtual images in real time. Maimone et al. [2014] designed wide field of view see-through glasses, which float virtual images on real-world. These devices provide 2D images or stereoscopic 3D images floated onto a specific plane.

Researchers working in 3D display fields have also been attracted to AR. Several methods to implement see-through 3D displays have been proposed. For instance, lens arrays for integral imaging were replaced by concave half mirror arrays [Hong et al. 2010] or by HOEs [Hong et al. 2014; Kim et al. 2015]. The concave half mirror arrays were implemented with index-matching oil, which gives transparency to conventional half mirror arrays. Takaki and Yamaguchi [2015] designed relaying optics that allow observers to see beyond a lens array. Waveguides were utilized for see-through integral floating displays [Hong et al. 2012] and for head-mounted 3D displays of projection-type [Javidi and Hua 2014]. All of these methods are based on the principles of integral imaging displays, which generate light fields by using a lens array.

Our work demonstrates how compressive light field displays can be extended to AR applications. For the present study, we implemented HOEs, which only diffuse incident light from a specific direction. The optical property of angular selectivity makes it possible to design see-through multi-layered displays (additive light field displays). Transparent multi-layered displays present new possibilities in this field, because they convey high transparency while maintaining image fidelity. Also, the proposed system can impart motion parallax to observers. We expect that the additive light field display can be modified for use with a head-mounted display.

### 3 Additive Light Field Displays

This section describes the principles of additive light field displays. First, we introduce diffuser HOEs (DHOEs) for see-through projection screens. Descriptions of the recording and reconstruction processes for DHOEs are presented. Second, we describe how light fields from DHOE layers are merged into a single light field by addition. The light fields from the layers do not interfere with each other since the illumination sources are incoherent. Third, we describe how an additive light field display expresses an optimized light field with a non-negative least squares method. We conclude by solving an optimization problem and presenting simulation results.

#### 3.1 Diffuser Holographic Optical Elements

##### 3.1.1 Recording Process

Figure 2 shows the recording scheme of DHOEs. Three coherent light sources are used for the full-color expressions of red, green, and blue. Neutral density filters are placed in front of the three light sources in order to modulate power densities, which affect the diffraction efficiency of each color in the reconstruction process. The three laser beams are merged into a single laser beam using dichromatic mirrors. The exposure time of the single laser beam is controlled by an electric shutter. Then, the laser beam is converted to the collimated plane wave using a spatial filter and a collimating lens. The laser beam is divided into signal and reference waves using a beam splitter. Mirrors guide the signal and reference waves to the holographic material. The signal wave passes a diffuser before it is introduced to the holographic material, while the reference wave is introduced directly to the holographic material.

Details of the recording are described on the right side of Fig. 2. The signal wave is converted to a scattered wave from the collimated plane wave via passage through the diffuser. The reference wave is introduced on the holographic material from the opposite side. Since the signal and reference waves are coherent, the two waves create an interference pattern on the holographic material. The interference pattern is recorded in the holographic material as a form of volume grating via refractive index modulation. Utilizing photopolymers that have definite optical transparency as holographic materials, we fabricate transparent projection screens. The details of the principles for the recording of the HOEs are presented in Supplementary Appendix A.

##### 3.1.2 Reconstruction Process

In the reconstruction process, the DHOE converts an incident wavefront to that of a signal wave (diffusing wave) when the Bragg con-
The Bragg condition is satisfied if a probe wave is projected on the DHOE with a wavefront identical to the reference wave of the recording process. The details of the principles of the Bragg condition are described in Supplementary Appendix A. In the Bragg condition, the wavefront of the probe wave is scattered by diffraction since a diffusing wavefront is recorded as the signal wave. The scattered wave is referred to as a reconstructed wave whose wavefront is identical to the signal wave. The amplitude of the probe wave, which corresponds to the projected image information, is preserved after diffraction. Thus, the image is displayed on the DHOE plane, which means the DHOE functions as a projection screen under the Bragg condition.

![Diagram of DHOE system](image)

**Figure 3:** The reconstruction process of DHOEs as illustrated with a 1-layer DHOE system. The reconstruction system consists of collimating and screen parts. Details of the collimating optics (a) and DHOE screen (b) are described. (b1) and (b2) indicate the angular selectivity and transparency of the DHOE screen, respectively.

The reconstruction system using beam projectors with a DHOE layer is shown in Fig. 3. First, white illumination from a backlight source is divided into three primary colored beams (red, green, and blue) using dichroic mirrors. Each colored beam is modulated by a separate LCD panel. Then, a dichroic prism combines and guides the three colored beams into relay optics. The combined beam passes through collimating optics, which convert the beam to a collimated plane wave. The function of the collimating optics also includes noise filtering and beam expanding. Finally, the collimated plane beam is introduced to the DHOE in an appropriate angle and is diffused at the DHOE plane. The displayed images are updated by the beam projector in a manner of conventional beam projection systems.

If a probe wave, which does not satisfy the Bragg condition, is introduced to the DHOE, the probe wave passes through the DHOE without diffraction, as illustrated in Fig. 3(b1). Figure 3(b2) describes a real-world scene passing through the DHOE without diffraction, since the Bragg condition is not satisfied. This property allows us to apply the DHOE as a transparent projection screen. The DHOE shows transmittance that is superior to half mirrors, which partially transmit and reflect a real-world scene. In addition, we are able to design the probe wave to diffuse at a desired depth for the DHOE screen. The details of the principles of the Bragg condition is satisfied. The Bragg condition is satisfied if a probe wave 1 and 2 are incident on the 2-layer DHOEs with different angles. The incident angles of probe waves 1 and 2 are \( \theta_1 \) and \( \theta_2 \), respectively. Since DHOEs 1 and 2 have different angular selectivity, probe wave 1 is diffused by DHOE 2 while it passes through the DHOE 1. On the other hand, probe wave 2 is scattered only by DHOE 1. Consequently, DHOEs 1 and 2 function as rear and front projection screens, respectively, for compressive displays. However, in practical application, probe waves 1 and 2 are weakly diffracted by DHOEs 2 and 1, respectively, which is referred to as a ghost effect. The reconstructed waves from the DHOE layers are merged into a single wave. Since the waves are from incoherent illumination sources, it can be described using the following equation:

\[
I_{\text{total}} = I_1 + I_2,
\]

where \( I_{\text{total}} \) is the intensity distribution of the merged wave, and \( I_1 \) and \( I_2 \) are the intensity distributions of the reconstructed waves from DHOEs 1 and 2, respectively.

**Figure 4:** The principle of the additive light field display is illustrated with a stack of two DHOE layers. Each layer generates independent additive light fields, which are merged into a single light field.

### 3.2 Additive Light Fields

#### 3.2.1 Incoherent Addition of Light Fields

Figure 4 describes how the additive light field displays work with multi-layer DHOEs. Each DHOE is fabricated via the recording process described in Section 3.1.1. In the recording processes of DHOEs 1 and 2, the incident angles of the reference waves are \( \theta_1 \) and \( \theta_2 \), respectively. In the reconstruction process, probe waves 1 and 2 are incident on the 2-layer DHOEs with different angles. The incident angles of probe waves 1 and 2 are \( \theta_1 \) and \( \theta_2 \), which are identical to reference waves 1 and 2, respectively. Since DHOEs 1 and 2 have different angular selectivity, probe wave 1 is diffused by DHOE 2 while it passes through the DHOE 1. On the other hand, probe wave 2 is scattered only by DHOE 1. Consequently, DHOEs 1 and 2 function as rear and front projection screens, respectively, for compressive displays. However, in practical application, probe waves 1 and 2 are weakly diffracted by DHOEs 2 and 1, respectively, which is referred to as a ghost effect. The reconstructed waves from the DHOE layers are merged into a single wave. Since the waves are from incoherent illumination sources, it can be described using the following equation:

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### 3.2.2 Parameterization for Additive Light Fields

In 4D light fields, rays in 3D space are parameterized by a pair of points on two parallel planes. A light field \( L(P_x, P_y, Q_x, Q_y) \) which passes through two points, \( P \) and \( Q \), which are located on the DHOE 1 and DHOE 2 planes, respectively. If the probe waves 1 and 2 are projected on the 2-layer DHOEs, a couple of the rays corresponding to the light field \( L(P_x, P_y, Q_x, Q_y) \) are reconstructed independently at points \( P \) and \( Q \). Thus, the light field \( L(P_x, P_y, Q_x, Q_y) \) is given using the following equation:

\[
L(P_x, P_y, Q_x, Q_y) = I_1(P_x, P_y) + I_2(Q_x, Q_y),
\]

where \( I_1 \) and \( I_2 \) are intensity distributions of the diffused images from DHOEs 1 and 2, respectively. When more than two DHOE layers are stacked for an additive light field display, it is more straightforward to parameterize light fields with a point on the \( xy \) plane and two angles. The two angles \( \theta_x \) and \( \theta_y \) are defined as follows:

\[
\theta_x = \tan^{-1} \frac{r_x}{r_z}, \quad \theta_y = \tan^{-1} \frac{r_y}{r_z}, \quad |\theta_x|, |\theta_y| \leq \theta_d
\]

where \( \vec{r} = (r_x, r_y, r_z) \) denotes a direction vector of a ray, and \( \theta_d \) is the diffusing angle of the DHOE layers. The 4D light field,
which is generated from the $n$-layer of the DHOEs, is given using the following equation:

$$L(x, y, \theta_x, \theta_y) = \sum_{i=1}^{n} I_i(x + h_i \tan \theta_x, y + h_i \tan \theta_y),$$

(4)

where $I_i$ is the intensity distribution of the diffused image from the $i$-th DHOE layer, and $h_i$ indicates the depth of the $i$-th DHOE plane, as described in Fig. 4. Equation (4) is based on the assumption that the scattering profile of the recorded diffuser is uniform.

### 3.3 Non-Negative Least Squares Problem

![Figure 5: A method that can be used to express the least squares problem by matrices is illustrated with 2D light fields and a couple of 1D additive layers. The column vectors of the target light fields and parameters are described on the left and right-hand sides, respectively. Each row of the projection matrix is determined by the specifications of target light fields and by the structure of the layers.](image)

For the additive light field display, we are compelled to solve the following problem:

$$\min \| L_t(x, y, \theta_x, \theta_y) - L(x, y, \theta_x, \theta_y) \|,$$

(5)

where $L_t(x, y, \theta_x, \theta_y)$ is the target light field, which is desired to be emitted from the additive light field display. If the parameters for the light field are assumed to be discrete, the light field could be represented as a 4D matrix. The matrix can be reshaped to a column vector $L_t$ with $M$ elements, as shown in Fig. 5. The intensity distributions from the $n$-layer DHOEs are also represented as a column vector $I$ with $N$ elements. We define the projection matrix $P$ as a binary matrix with $N$ columns and $M$ rows. When the projection matrix is multiplied by the column vector $I$, the elements of the column vector $I$ are selectively summed for each row. Thus, the projection matrix can be designed to satisfy the following equation, which corresponds to Eq. (4):

$$L_t = \sum_{j=1}^{N} P_{ij} I_j,$$

(6)

where $L_t$ denotes the additive light field $L_t(i; x, y, \theta_x, \theta_y)$, and $P$ is the projection matrix. Consequently, Eq. (5) can be rewritten with the column vectors and matrix as follows:

$$\min \| L_t - PI \|.$$

(7)

Finding an optimized solution for Eq. (7) is to solve linear the least squares problem. However, there is a constraint whereby the elements of the column vector $I$ should be positive since the intensity of illumination cannot be less than zero. Hence, a non-negative least squares problem is encountered. The trust region algorithm [Coleman and Li 1996] is applied to solve the problem. Figure 6 demonstrates the simulation results of the introduced algorithm.

### 4 Implementation

In this section, we describe the implementation process of the additive light field display prototype. First, a description is presented for the recording process of the DHOEs. Second, a reconstruction system for additive light fields is demonstrated. Detailed configurations of recording and reconstruction systems are described in Supplementary Appendix B. In the last of this section, we introduce the software that is used for the prototype.

#### 4.1 Recording System

To record a full-color DHOE, we apply a wavelength multiplexing method [Coufal et al. 2000]. Three different lasers are used for the coherent illumination sources with three colors: red (Cobolt Flamenco, 660 nm), green (Coherent Verdi, 532 nm), and blue (Spectra-Physics Excelsior, 473 nm). Photopolymers (HX film) provided from Covestro AG are utilized as the holographic material. The diffuser is a holographic diffuser from Edmund with a diffusing angle of $10^\circ$. The signal wave passes through the diffuser and is introduced to the photopolymer in the normal direction. The reference wave is guided by two mirrors and introduced to the photopolymer. The incident angles of the reference waves for recording DHOEs 1 and 2 are $45^\circ$ and $60^\circ$, respectively. The different incident angles are set in order to reduce the ghost effect. The exposed energies of the red, green, and blue lasers are 36 mJ/cm$^2$, 10.92 mJ/cm$^2$, and 32.4 mJ/cm$^2$, respectively. Table 1 describes the specifications of the recording process for the DHOEs.

#### 4.2 Reconstruction System

For the additive light field display, we use PT-AE1000E beam projectors that support a 1920×1080 spatial resolution with scanning frequencies 30-70 kHz and 50-87 Hz for horizontal and vertical directions, respectively. The relay optics are replaced by inverted projection lenses in the beam projectors. Two 0.16×silver series tele-centric lenses from Edmund function are used as the collimating and noise filtering optics. The projection beam-size, which is determined by the tele-centric lens, is 48×27 mm$^2$. The collimated projection beams are guided to the DHOEs by mirrors as probe waves. The probe waves 1 and 2 are introduced to the DHOEs at angles of $45^\circ$ and $60^\circ$, respectively. The DHOE screen has a circular form with a diameter of 37 mm. The prototype of the additive light field display has a viewing window of $30\times22.5$ mm$^2$ with 692×900 pixels that provide 595 dpi under ideal conditions. In practical, the dpi is 406, which reflects a decline of 32% caused by the optical blur of the collimating system. The gap between the DHOE layers is set at 6 or 10 mm. The gaps show the flexibility of the system design, which is achieved with low diffraction of the DHOE layers. The 10 mm thickness system (prototype A) shows a 3D image of a statue of Flamenco, 660 nm), green (Coherent Verdi, 532 nm), and blue (Spectra-Physics Excelsior, 473 nm). Photopolymers (HX film) provided from Covestro AG are utilized as the holographic material. The diffuser is a holographic diffuser from Edmund with a diffusing angle of $10^\circ$. The signal wave passes through the diffuser and is introduced to the photopolymer in the normal direction. The reference wave is guided by two mirrors and introduced to the photopolymer. The incident angles of the reference waves for recording DHOEs 1 and 2 are $45^\circ$ and $60^\circ$, respectively. The different incident angles are set in order to reduce the ghost effect. The exposed energies of the red, green, and blue lasers are 36 mJ/cm$^2$, 10.92 mJ/cm$^2$, and 32.4 mJ/cm$^2$, respectively. Table 1 describes the specifications of the recording process for the DHOEs.

Figure 7 shows photographs of the additive light field display prototype. We used various 3D images in order to exhibit the versatility of the additive light field display. The first is a 3D image of a statue of Mercury with minute details. In that result, minute details such as the folds of clothing or locks of hair are expressed. The second is a 3D image of two pair of dice with different colors and depths. Full-color and occlusion effect between the dice can be observed. The third includes 3D images of a toy plane, tropical fish, and a Chinese dragon, which confirms the generality for types of 3D images.
Additive light field display
Optimized additive layers

5 Evaluation

In this section, the prototype of the additive light field display is evaluated. First, we discuss the valuation of the DHOE layers as see-through projection screens. Diffraction efficiency and transmittance of the DHOE layers are demonstrated. Second, we assess the display performances of the additive light field display. The peak signal-to-noise ratios (PSNRs) of the additive light field display, the attenuation-based display, and the polarization field display are demonstrated and compared via simulation.

5.1 Diffraction Efficiency and Transmittance

The diffraction efficiency of the DHOE is defined as the ratio of the reconstructed wave intensity to the probe wave intensity. Thus, the diffraction efficiency is an important factor in determining the brightness and energy efficiency of the additive light field display. To measure the diffraction efficiency, a probe wave is guided to the DHOE with a proper incident angle, and then a reconstructed wave is generated, as described in Section 3.1.2. In order to measure the reconstructed wave intensity, an optical power meter is placed in front of the DHOE. The diffraction efficiencies for the red, blue, and color illuminations are measured with the corresponding wavelengths of the probe waves. The results of the experiment are shown in Table 2.

The transmittance of the DHOE is defined as the ratio of the transmitted wave intensity to the incident wave intensity. High transmittance is essential to provide a bright real-world scene or to stack multiple DHOE layers. In order to measure the transmittance of natural illumination sources, we use a white light source and an optical spectrometer. The transmittance of DHOE layer 1 according to the wavelength is demonstrated in Fig. 8. There are three dips of transmittance since the recorded gratings of the three colors affect the transmittance. The average transmittance of the DHOE layer is 0.91, which is high enough to be seen beyond the display system. Figure 9 demonstrates the high degree of transparency of the DHOE layers. The SIGGRAPH logo is projected on the front.
Figure 7: These photographs of the prototype additive light field display show several reconstructed light field images from the two different systems. The first three images (Mercury, toy plane, and dices) are the results of the additive light field display with a 10 mm gap (prototype A). The other two images (tropical fish and Chinese dragon) are the results of the additive light field displays with a 6 mm thickness (prototype B). The original light fields, additive layer images, and detailed parallax views are illustrated with the enlargements of the results. Additional results and sources of the 3D images are included in Supplementary Appendix C.
DHOE layer. A car, a house, and a lovely couple are real objects behind the additive light field display.

<table>
<thead>
<tr>
<th>Incident angle of the probe wave</th>
<th>45°</th>
</tr>
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<tbody>
<tr>
<td>Diffraction efficiency</td>
<td></td>
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<tr>
<td>473 nm</td>
<td>11.1 %</td>
</tr>
<tr>
<td>532 nm</td>
<td>5.8 %</td>
</tr>
<tr>
<td>660 nm</td>
<td>23.3 %</td>
</tr>
</tbody>
</table>

The DHOE layer prototype shows a different diffraction efficiency according to the three wavelengths since the exposed power of each laser is different in the recording process, as described in Section 4.1. Since uniform diffraction efficiency does not guarantee optimized color reproducibility, we manually balanced the diffraction efficiency ratio between each color. However, color artifacts are still observed in the prototype. A rigorous optimization process for the diffraction efficiency ratio is essential to performing an accurate full-color expression. Based on the colorimetry, the optimized diffraction efficiency ratio of the DHOE layer could be determined by the spectral density of the illumination that is utilized as probe waves.

The transmittance of the DHOE layer is demonstrated. There are three dips, (471, 0.76), (532, 0.81), and (654, 0.69), which correspond to the recorded wavelengths.

5.2 PSNR of Additive Light Field Displays

The three methods for compressive displays show different display performances according to utilized algorithms as shown in Fig. 10. We compute the PSNRs in order to assess display performances. The PSNRs are estimated according to the differences between the target light fields and the reconstructed light fields. In the simulation, we neglect other technical issues such as the attenuation of brightness by a stack of layers. Figure 10 describes the average PSNRs for the five target light fields. The green, orange, and blue lines correspond to the additive light field display, attenuation-based display, and polarization field display, respectively. The solid, dotted, and broken lines indicate PSNRs of reconstructed light fields derived by trust region [Coleman and Li 1996; Wetzstein et al. 2011], simultaneous algebraic reconstruction technique (SART) [Andersen and Kak 1984; Lanman et al. 2011], and factorization algorithm [Lee and Seung 1999; Hirsch et al. 2014], respectively. According to the simulation results, the additive light field display shows an intermediate display performance from among the three compressive display methods. When the trust region or SART algorithm is utilized, the additive light field display surpasses the attenuation-based display. In the attenuation-based display, a halo effect appeared since the logarithm transformation amplifies errors. The halo effect is diminished when a factorization algorithm is applied that does not include the logarithm transformation. In that case, the attenuation-based display shows a performance that is similar to that of the additive light field and polarization field displays.

6 Analysis

In this section, we describe two issues of compressive displays. First, the validity of ray approximation for compressive displays is analyzed with wave optics. Since the SLM consists of pixelated structures, the diffraction effect could be considerable under specific conditions. When the diffraction effect could not be neglected, a method to include the diffraction effect in the optimization algorithm is described. Second, the inability to express uncorrelated
target light fields is analyzed using a statistical method. The analytical estimation is confined to the additive light field display.

### 6.1 Validity of Ray Approximation

When a plane wave passes through a rectangular aperture, the wave is diffracted. In the Fraunhofer approximation, the diffraction effect is described as follows [Saleh et al. 2007].

\[
I_i(u, v) = I_0 \sin^2 \left( \frac{w x u}{\lambda d} \right) \sin^2 \left( \frac{w v u}{\lambda d} \right),
\]

(8)

where the rectangular aperture is placed at the origin point (0, 0); \(I_i(u, v)\) is the intensity of the diffracted wave at \((u, v, d)\); \(I_0\) is the intensity of the incident plane wave; \(w_x\) and \(w_y\) are the width and height of the rectangular aperture, respectively; and, \(\lambda\) and \(d\) are the wavelength and propagation distance of the wave, respectively. If \(d\) is sufficiently large compared with \(u\) or \(v\), Eq. (8) can be approximated as follows:

\[
I_i(\theta_x, \theta_y) \approx I_0 \sin^2 \left( \frac{w x u}{\lambda} \theta_x \right) \sin^2 \left( \frac{w v u}{\lambda} \theta_y \right),
\]

(9)

where \(\theta_x\) and \(\theta_y\) are \(\tan^{-1}\left( u/d \right)\) and \(\tan^{-1}\left( v/d \right)\), respectively.

Since the SLM consists of pixelated rectangular apertures, each incident wave is diffracted at each pixel, as described above. This diffraction effect, which is not considered in the optimization, could degrade the image fidelity of compressive displays. For instance, the diffraction effect in compressive displays with 2-layer SLMs is analyzed as follows. First, a local plane wave from a pixel \(P(x_p, y_p)\) of the rear SLM is considered, which is approximated to a ray \(L(x_p, y_p, \theta_x, \theta_y)\) in a 4D light field. When the local plane wave passes through a pixel \(Q(x_q, y_q)\) of the front SLM, the wave is diffracted by the rectangular aperture of the pixel \(Q\). The diffracted wave is divided into local plane waves, which can be approximated to a set of rays. Each ray passes the pixel \(Q\) with altered propagation angles \(\theta'_{x}\) and \(\theta'_{y}\). Tracing back the optical path of the rays, the rays are parameterized in a 4D light field domain.

\[
x'_p = x - \Delta h \tan \theta'_{x} = x_p - \Delta h \tan (\theta_x - \tan \theta'_{x}),
\]

(10)

\[
y'_p = y = \Delta h \tan \theta'_{y} = y_p + \Delta h \tan (\theta_y - \tan \theta'_{y}),
\]

(11)

where \(\Delta h\) is the gap between SLM layers, and \(x'_p\) and \(y'_p\) are the 4D light field parameters of rays \(L_q(x'_p, y'_p, \theta'_{x}, \theta'_{y})\). According to Eq. (9), the intensity of each ray is given as follows:

\[
I(L; \theta'_{x}, \theta'_{y}) \approx I_0 \sin^2 \left( \frac{w x u}{\lambda} (\theta_x - \theta'_{x}) \right) \sin^2 \left( \frac{w v u}{\lambda} (\theta_y - \theta'_{y}) \right).
\]

(12)

If we assume that rays with intensities of less than 0.25\(I_0\) could be neglected, the set of rays covers an ellipse on the rear SLM plane. The major or minor axis of the ellipse is given by the following equation:

\[
a = \max_{\theta'_{x}} |x'_p - x_p| \approx \Delta h \max_{\theta'_{x}} |\theta'_{x} - \theta_x| = \Delta h \frac{\lambda}{w_x} c_{0.5},
\]

(13)

\[
b = \max_{\theta'_{y}} |y'_p - y_p| \approx \Delta h \max_{\theta'_{y}} |\theta'_{y} - \theta_y| = \Delta h \frac{\lambda}{w_y} c_{0.5},
\]

(14)

where either \(a\) or \(b\) is the major or minor axis of the ellipse, and \(c_{0.5}\) is a constant which is \(\sin^{-1}(0.5)\). Since approximation for the optimization is valid when each axis is smaller than a pixel pitch, the validity condition is described as follows.

\[
v_x = \frac{a}{w_x} \approx \frac{\Delta h c_{0.5} \lambda}{w_x^2} < 1,
\]

(15)

\[
v_y = \frac{b}{w_y} \approx \frac{\Delta h c_{0.5} \lambda}{w_y^2} < 1,
\]

(16)

where \(v_x\) and \(v_y\) are validity coefficients. Consequently, if a pixel pitch is much smaller than the distance to the panel, the ray approximation could fail, as described in Fig. 11.

![Figure 11: A validity curve for the ray approximation is described. The wavelength of illumination is set to 660 nm, to show the largest diffraction. Ray approximation is not guaranteed in an LCD layered structure with our prototype specifications (vertical dpi), while it has validity in the other previous prototypes. In order to achieve a high dpi with conventional display panels (e.g. LCD layers), the diffraction effect should be considered.](image)

This problem could be alleviated if the diffraction effect is included in the projection matrices. An example would be a more specific system, with valid coefficients, \(v_x\) and \(v_y\), of 2. In this case, the adjacent 25 pixels could generate the same light fields with different intensities. Thus, the modified light field expression is given using the following equation:

\[
L_p = \sum_{j=-2}^{2} \sum_{i=-2}^{2} k_{ij} t_1(x_p + iw_x, y_p + jw_y) t_2(x_q, y_q),
\]

(17)

where \(L_p\) indicates a ray \(L(x_p, y_p, \theta_x, \theta_y)\); \(k_{ij}\) is the intensity coefficient derived from Eq. (12); and, \(t_1(x, y)\) and \(t_2(x, y)\) denote the spatially varying transmittance functions of the rear and front SLM layers, respectively. Based on Eq. (17), we could revise the projection matrix for the rear layer. The simulation results are shown in Fig. 12, verifying the analysis of the diffraction effect. However, the optimization time is sacrificed since the number of non-zero elements of the projection matrix is increased approximately 25-fold.

### 6.2 Uncorrelated Target Light Fields

The additive light field display is limited in its generation of uncorrelated target light fields. The attenuation-based display has a solution to utilize a front layer as a parallax barrier or pinhole array, but the additive light field and polarization field displays do not have an efficient solution. This problem was described in examples from Lanman et al. [2011]. In the present study, we approach the problem with a statistical method, which enables a more quantitative analysis. First, we assume that uncorrelated target light fields consist of independent and identically distributed random variables. We compute expected values of mean squared errors (MSE) with a random target light fields. The constraints of the least squares problems are neglected in order to determine analytic solutions. Without
the constraints, the solutions are given by the following equation:

$$\min \| y - Px \|, \quad (18)$$

$$x^* = PP^+ y, \quad (19)$$

where $x$ and $y$ indicate parameters and a target light field, respectively; $x^*$ is the solution of the least squares problem; and $P^+$ is the pseudoinverse of the projection matrix $P$. Thus, errors in the solution are given by the following equation:

$$error = (I - PP^+)y = C_{n \times n}y, \quad (20)$$

where $I$ is an $n \times n$ unit matrix, and $C$ is an $n \times n$ matrix defined as $C = I - PP^+$. Using Eq. (20), we derive the expected value of MSE, as follows:

$$E\{MSE\} = \frac{1}{n} E \left[ \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} y_{ij} \right]^2 = \frac{1}{n} \sum_{i=1}^{n} (\mu_i^2 + \sigma_i^2), \quad (21)$$

where $\sigma_i = \sigma(y)(\sum_{j=1}^{n} C_{ij}^2)^{\frac{1}{2}}$ and $\mu_i = E(y) \sum_{j=1}^{n} C_{ij}$. The details of the derivations are described in Supplementary Appendix D.

According to Eq. (21), the expected value of the MSE for the additive light field display is estimated, and is shown in Fig. 13. The details of the specifications that are used to design the projection matrix are described in Supplementary Appendix D.

The blue line and orange dots in Fig. 13 indicate the expected and sampled PSNRs of the additive light field display according to the compression ratio. The sampled PSNRs are average values, which are derived by the trust-region algorithm with constraints from 30 sets of uncorrelated target light fields.

**Figure 12:** Diffraction effect when the validity constants are 2. The 2D (eye chart) and 3D images are blurred when diffraction is considered. When a compensated projection matrix is applied, as described in Eq. (17), the reconstructed images are improved.

**Figure 13:** Expected values of the MSE of the additive light field display without constraints are described. Logarithms of the expected values are utilized as expected PSNRs. For comparison, the average PSNRs derived via iteration algorithm with constraints are also estimated from 30 sets of uncorrelated target light fields.

### 7 Discussion

#### 7.1 Benefits and Limitations

Multi-layered Displays for AR. The additive light field display is a novel type of compressive light field display that is based on addition rather than on either multiplication or rotating polarization. DHOEs are utilized as transparent projection screens for additive light field displays. Thus, an additive light field display has the potential to be applied to AR. The multi-layered structure of DHOEs has several merits for compressive displays. Issues such as diffraction, brightness, color-channel cross talk, and moiré, which appear in the use of stacked LCDs, are significantly improved with multi-layer DHOEs. Hence, the additive light field display with DHOEs could be an effective method to realize compressive displays with high dpi.

In the present study, we implement a prototype of the additive light field display. High fidelity in reconstructed light fields is demonstrated in the display results. Full-color expression is realized with full-color DHOE layers. The pixel density of the additive light field display prototype is estimated at 406 dpi, which is about 5 times higher than moderate LCD panels. The average transparency of each DHOE layer is about 90%, which is superior to other beam combiners such as beam splitters and waveguides. The additive light field display shows a display performance that is similar to that of the polarization field and the attenuation-based displays. We analyze the diffraction effect of a stack of SLM layers as well as the validity of the ray approximation for compressive displays. According to the analysis, the ray approximation is valid only for the additive light field display in instances where a high pixel density is essential.
Ghost Effect  The experimental results of the prototype show a ghost effect, which degrades the image fidelity. A ghost effect occurs due to an undesired diffraction of probe waves (Bragg mismatched reconstruction [Lee et al. 2016]). For instance, the front DHOE layer weakly diffracts a probe wave that is intended to be diffracted only by the rear DHOE layer. In addition, a probe wave reflected by the rear DHOE layer could also be diffracted by the front layer. A detailed analysis of the ghost effect is included in Supplementary Appendix E. In order to alleviate the ghost effect, we establish angular spacing between the incident angles of the probe waves. There are some methods that can be employed for further improvement. First, we can utilize anti-reflection glasses, or we could block the optical paths of reflected light with half-wave plates and polarizers. Second, it is also possible to use thicker photopolymers for recording DHOEs, which have a higher degree of angular selectivity.

Optical Blur  Since the collimating system of the prototype involves a chromatic aberration, color dispersion is observed in the display results. The color dispersion blurs the effective pixel pitch, leading to a degradation of the spatial resolution. Meanwhile, the imaged pixels are so small that the projection beam has a narrow depth of focus, which could be responsible for the blurred images. These artifacts, which we call optical blur, are demonstrated in Supplementary Appendix E. Because of the optical blur, the dpi of a projected image can decline from 595 to 406. The optical blur could be alleviated through several engineering efforts. First, free-form waveguides can function as collimating optics and compensators for chromatic dispersion. Second, laser projectors can replace beam projectors for extending the depth of focus.

More than 2 DHOE Layers  In this paper, we utilize a 1D angular selectivity of the DHOE layer. Nonetheless, the DHOE layer has a 2D angular selectivity. Thus, we could design a 4-layer system with projections from left, right, top, and bottom. However, the implementation of a prototype with 3 or more DHOE layers would involve more engineering issues. A display system for each layer would be too bulky to allow an increase in the number of layers. Optimizing the system size should precede an increase in the number of layers. Applying free-form waveguides and pico-projectors could simplify the system. Also, we could utilize a diverging wave for the reference wave in the recording process. The projection beams could then function as probe waves without collimating optics in the reconstruction process. Another engineering issue of the prototype is a small display window, which would attenuate the necessity of a multi-layered system. The display window could be enlarged using holographic printing methods.

Uncorrelated Light Fields  The additive light field display has an intrinsic limitation, which is analyzed in Section 6.2. This difficulty in compressing uncorrelated light fields is a common issue for the attenuation-based and polarization field displays. The attenuation-based display, however, is relatively free from this problem since the front SLM layer could form either a parallax barrier or a pinhole array. However, when a wide field of view is essential, the attenuation-based display has the same limitation in the absence of a time-multiplexing method.

Limited DOF & Applications  The light field spectrum that is emitted by additive light field displays forms a few lines in the frequency domain, which means the depth of field is restricted to the regions nearby the individual layers [Narain et al. 2015]. The limited depth of field could be a barrier for AR applications since virtual images are difficult to superimpose over real objects. However, it is able to apply additive light field displays as whiteboards, windshields, and in the simulated shooting game shown in Fig. 14. In addition, with accommodation cues, additive light field display systems have the potential to be modified for use with head- mounted displays. Indeed, head-mounted displays and AR screens using holographic materials are currently being developed by companies such as DigiLens and HoloPro.

7.2 Future Work

HOEs have the potential for the realization of AR. Applying HOEs to conventional display methods will be interesting topics for future work. For instance, it is feasible to apply HOEs to see-through head-mounted displays. HOEs could also function as a transparent projection screen for a compressive light field projection system [Hirsch et al. 2014]. Methods that include the diffraction effect in optimization algorithms could be applied to compressive displays with SLM layers. If the diffraction effect is not severe, the method could enhance the display performance of compressive light field displays.

The principles that govern the additive light field displays could be applied to conventional volumetric displays with additive layers such as multi-focal plane displays that use switchable lenses [Liu et al. 2008; Love et al. 2009]. A light field stereoscope display with additive layers has already been simulated by Huang et al. [2015]. Advanced computational techniques for AR could be applied to additive light field displays. For instance, the introduction of a direct 3D interactive system with a Kinect camera [Hilliges et al. 2012] could be an interesting application for an additive light field display. Computer vision algorithm that could recognize and classify real objects is another issue [Hagbi et al. 2011] to be explored.

8 Conclusion

We combined compressive light field displays, which is an emerging technology for light field displays, and HOEs. The compressive light field displays demonstrated a high level of display perfor-
manances in terms of brightness, resolution, and contrast in applications where conventional glasses-free 3D displays still suffer from the trade-off between spatial and angular resolutions. HOEs are versatile optical elements, which have transparency, angular selectivity, and an ability for spatial and wavelength multiplexing. In the present study, we applied the merits of HOEs to compressive light field displays. We implemented and utilized DHOEs as transparent additive projection screens for compressive displays. Additive light field displays enable observers to see real-world beyond a display in full-color, bright, and high-fidelity 3D images. With advances in computational technologies, additive light field displays with AR could be realized in the near future.

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References


