Homography, Mosaicing and Stabilization

Computer Vision

Some Slides are from Prof. Robert Collins, Penn State Univ.

Hierarchy of Transformations

$$\begin{bmatrix} A & b \\ c^T & 1 \end{bmatrix}$$ **projective**

↓

$$\begin{bmatrix} A & b \\ 0 & 1 \end{bmatrix}$$ **affine**

↓

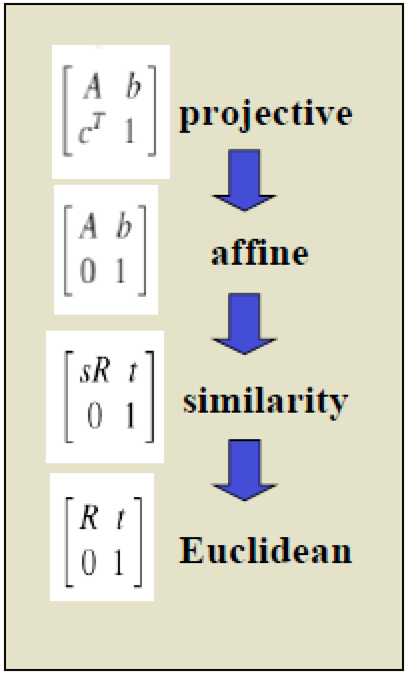
$$\begin{bmatrix} sR & t \\ 0 & 1 \end{bmatrix}$$ **similarity**

↓

$$\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$$ **Euclidean**

subgroup hierarchy

Composition in a Hierarchy



similarity * similarity = similarity

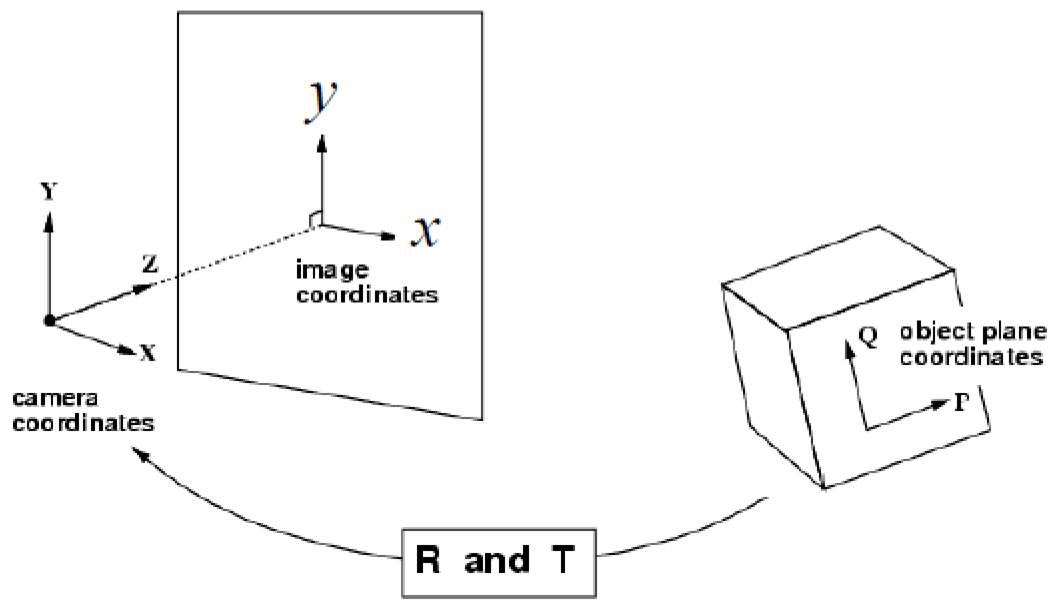
similarity * affine = affine

Euclidean * affine = affine

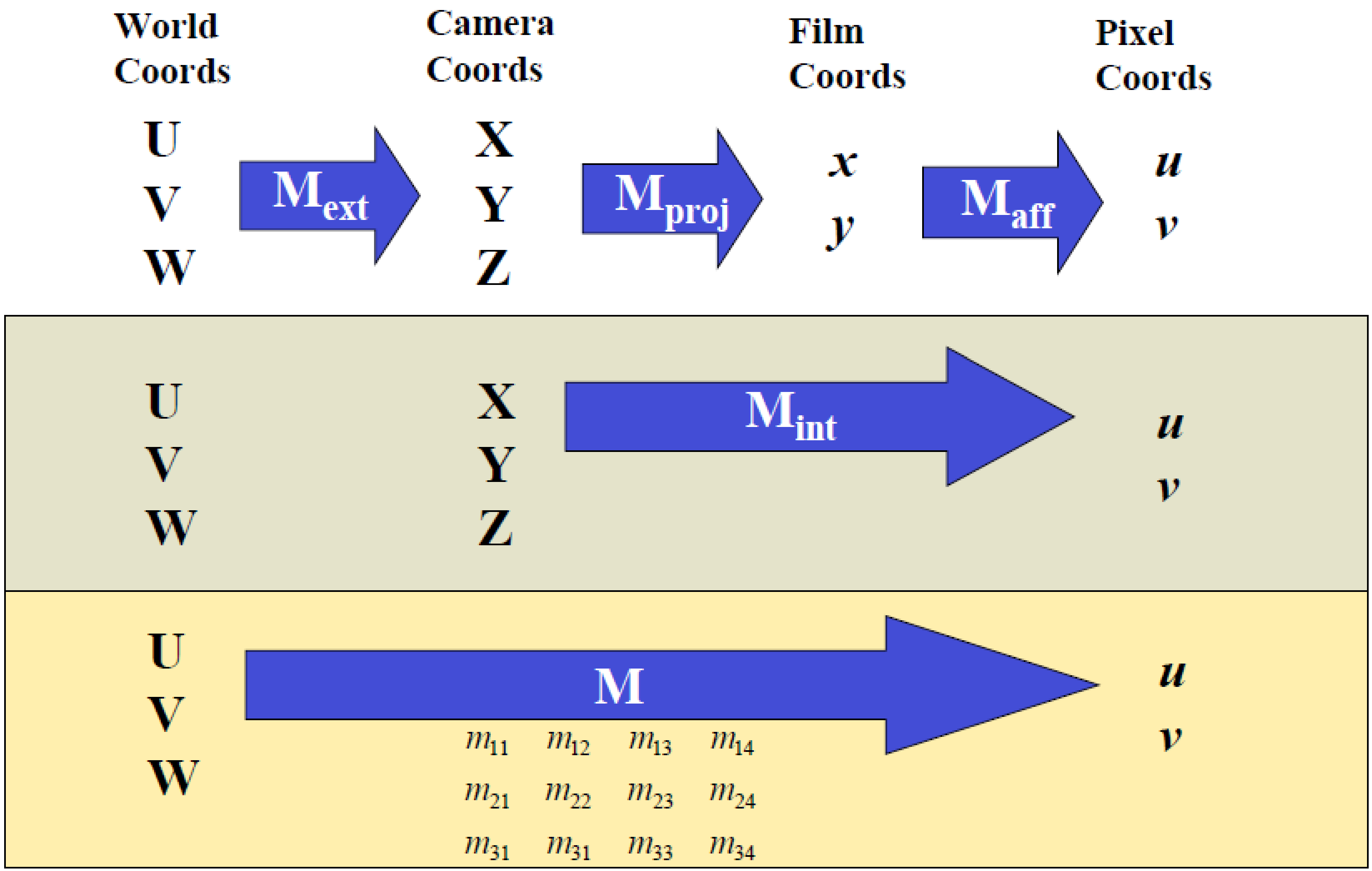
any * projective = projective

Why did we go through all this?

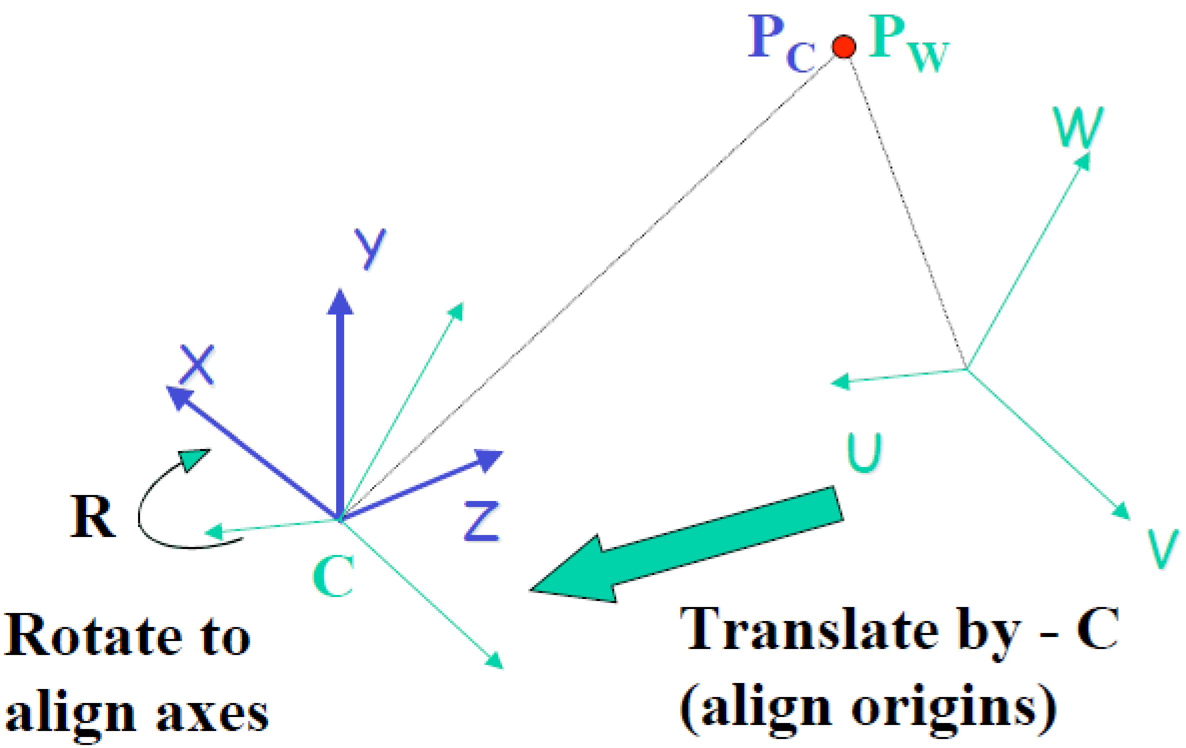
Projection of Points on Planar Surface



Review : Forward Projection



World to Camera Transformation



$$P_C = R\,(\,P_W - C\,)$$
$$= R\,P_W + T$$

Perspective Matrix Equation

(Camera Coordinates)

$$x = f\frac{X}{Z}$$

$$y = f\frac{Y}{Z}$$

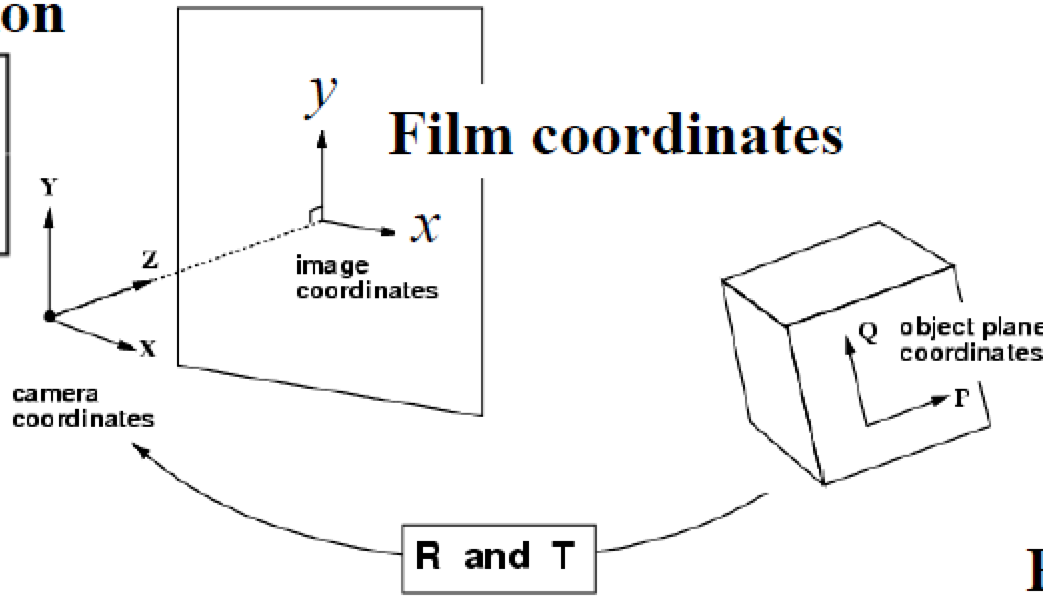
$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$p = M_{\text{int}} \cdot P_{\text{C}}$$

Projection of Points on Planar Surface

Perspective projection

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



Film coordinates

$$\begin{bmatrix} p \\ q \\ 0 \\ 1 \end{bmatrix}$$

Point on plane

Rotation + Translation

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Projection of Planar Points

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{bmatrix} \begin{bmatrix} p \\ q \\ 1 \end{bmatrix}$$

Projection of Planar Points (cont)

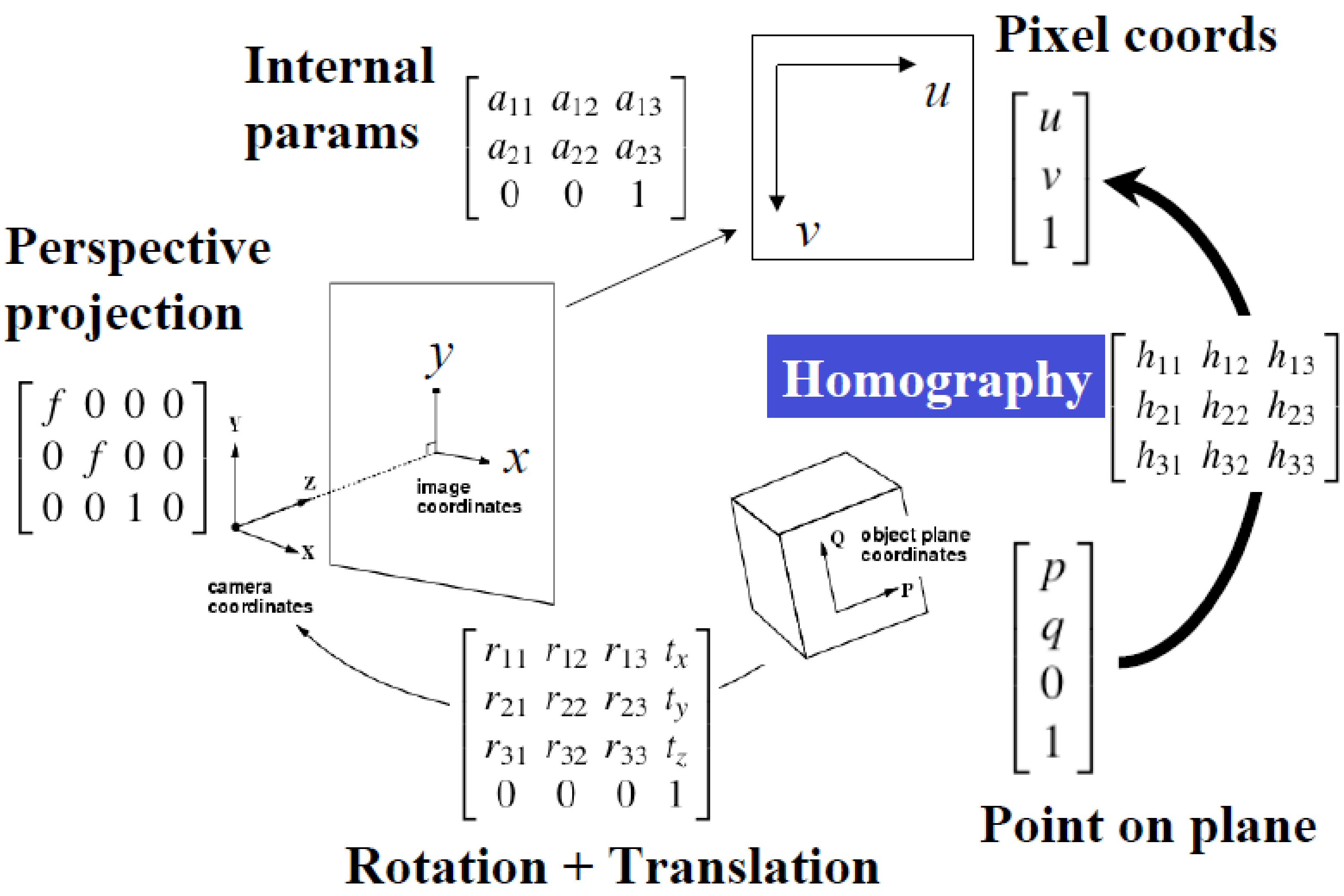
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ r_{31} & r_{32} & t_z \end{bmatrix} \begin{bmatrix} p \\ q \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \begin{bmatrix} fr_{11} & fr_{12} & ft_x \\ fr_{21} & fr_{22} & ft_y \\ r_{31} & r_{32} & t_z \end{bmatrix} \begin{bmatrix} p \\ q \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} p \\ q \\ 1 \end{bmatrix}$$

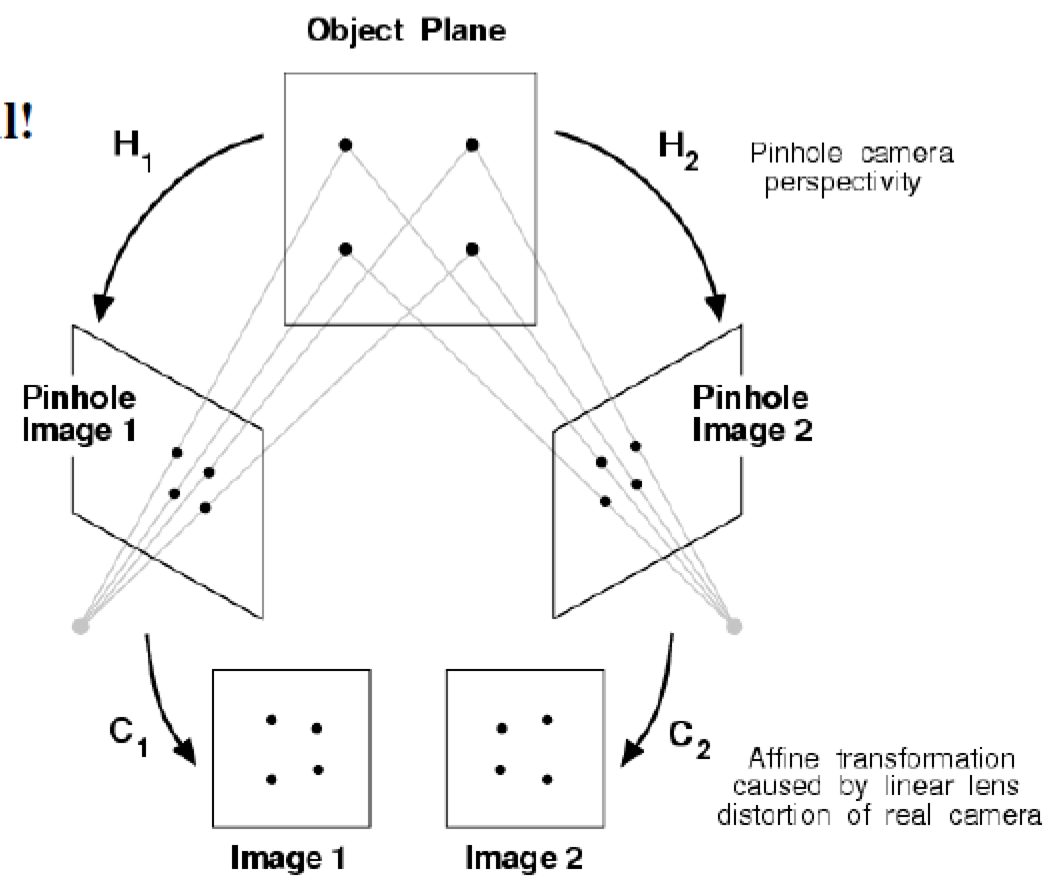
Homography H (planar projective transformation)

Overview: Planar Projection

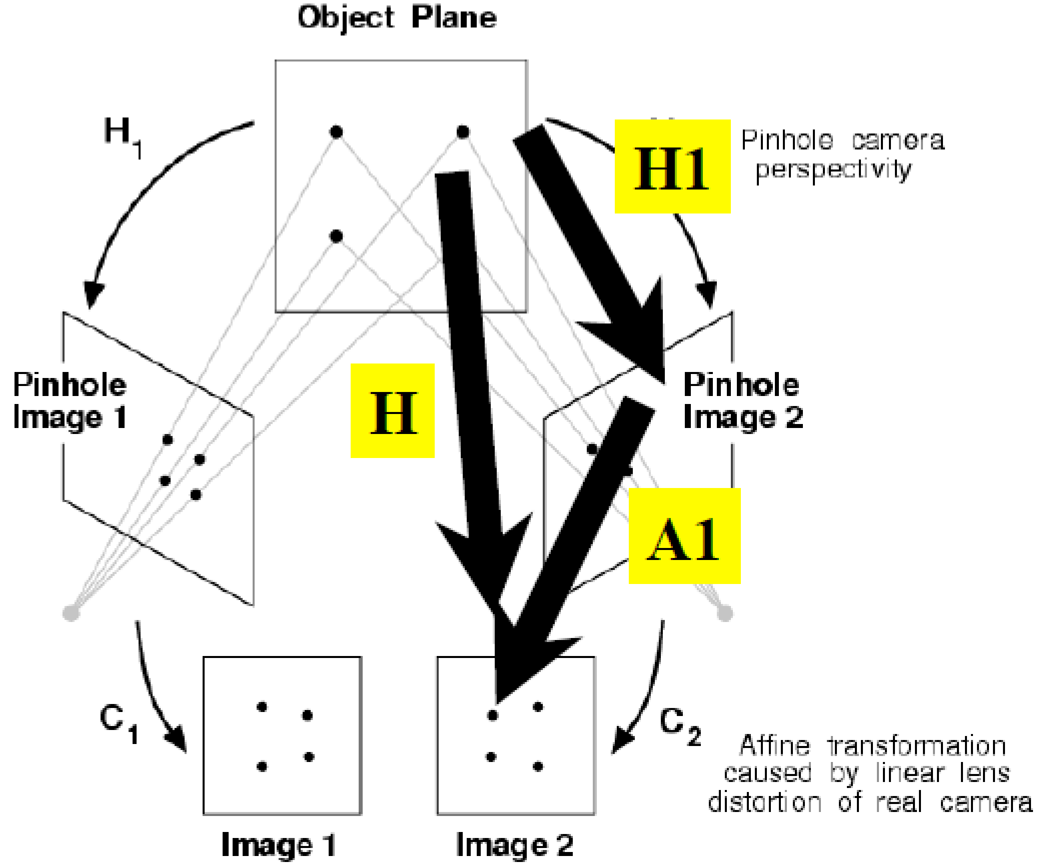


Planar Projection Diagram

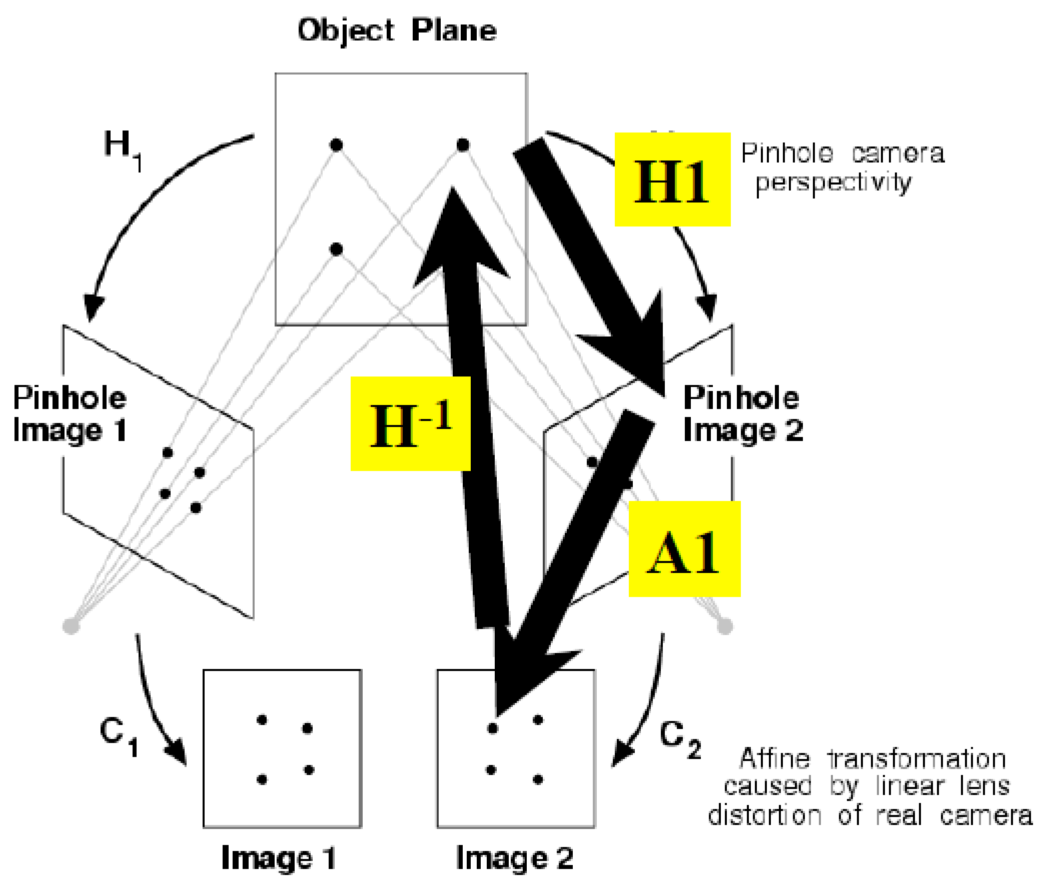
Here's where transformation groups get useful!



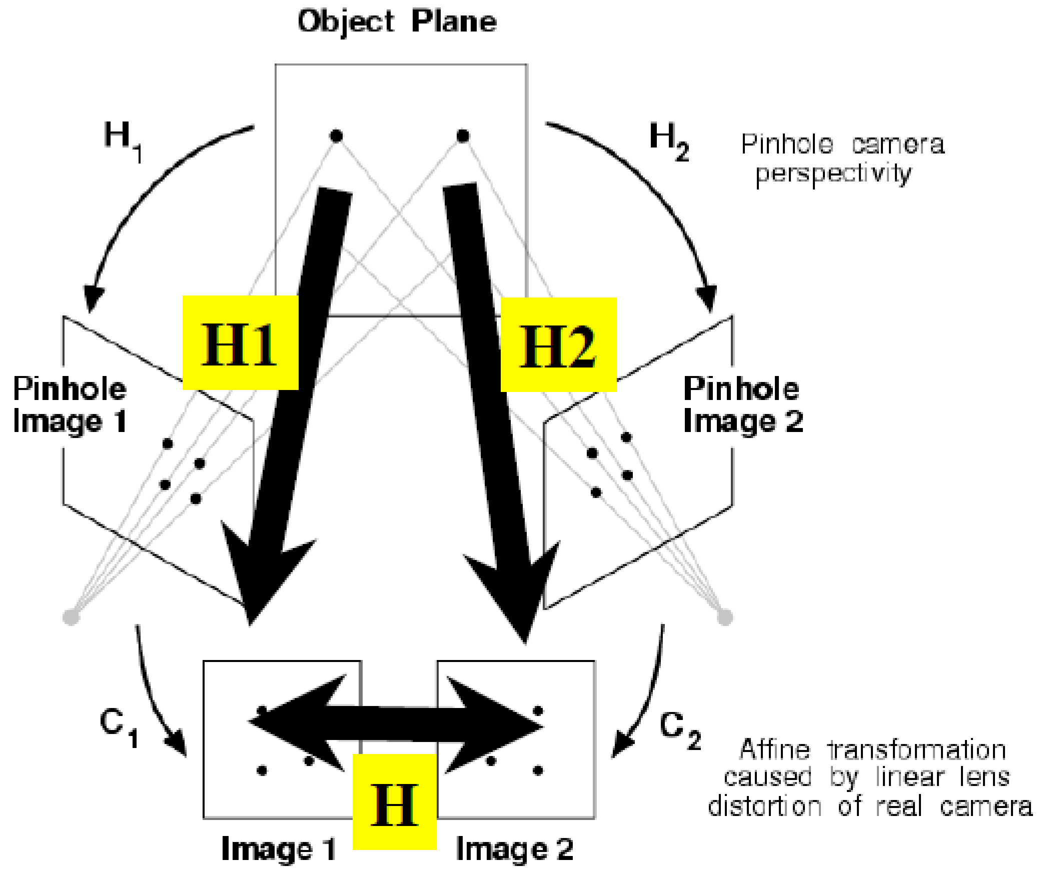
General Planar Projection



General Planar Projection



General Planar Projection



Estimating a Homography

Matrix Form:

Scale~

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Equations:

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

Degrees of Freedom?

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

There are 9 numbers $h_{11}, \ldots, h_{33}$, so are there 9 DOF?

No. Note that we can multiply all h_{ij} by nonzero k without changing the equations:

$$x' = \frac{kh_{11}x + kh_{12}y + kh_{13}}{kh_{31}x + kh_{32}y + kh_{33}} \qquad y' = \frac{kh_{21}x + kh_{22}y + kh_{23}}{kh_{31}x + kh_{32}y + kh_{33}}$$

$$\Rightarrow$$

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}} \qquad y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

Enforcing 8 DOF

One approach: Set $h_{33} = 1$.

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + 1}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + 1}$$

Second approach: Impose unit vector constraint

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

Subject to the constraint: $h_{11}^2 + h_{12}^2 + h_{13}^2 + h_{21}^2 + h_{22}^2 + h_{23}^2 + h_{31}^2 + h_{32}^2 + h_{33}^2 = 1$

L.S. using Algebraic Distance

Setting $h_{33} = 1$

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + 1}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + 1}$$

Multiplying through by denominator

$$(h_{31}x + h_{32}y + 1)x' = h_{11}x + h_{12}y + h_{13}$$

$$(h_{31}x + h_{32}y + 1)y' = h_{21}x + h_{22}y + h_{23}$$

Rearrange

$$h_{11}x + h_{12}y + h_{13} - h_{31}xx' - h_{32}yx' = x'$$

$$h_{21}x + h_{22}y + h_{23} - h_{31}xy' - h_{32}yy' = y'$$

Algebraic Distance, h_{33}=1 (cont)

2N x 8 **8 x 1** **2N x 1**

$$
\begin{array}{l}
\textbf{Point 1} \\ \\ \textbf{Point 2} \\ \\ \textbf{Point 3} \\ \\ \textbf{Point 4}
\end{array}
\begin{bmatrix}
x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x'_1 & -y_1x'_1 \\
0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y'_1 & -y_1y'_1 \\
x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x'_2 & -y_2x'_2 \\
0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y'_2 & -y_2y'_2 \\
x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3x'_3 & -y_3x'_3 \\
0 & 0 & 0 & x_3 & y_3 & 1 & -x_3y'_3 & -y_3y'_3 \\
x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4x'_4 & -y_4x'_4 \\
0 & 0 & 0 & x_4 & y_4 & 1 & -x_4y'_4 & -y_4y'_4
\end{bmatrix}
\begin{bmatrix}
h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32}
\end{bmatrix}
=
\begin{bmatrix}
x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \\ x'_4 \\ y'_4
\end{bmatrix}
$$

additional points $\vdots$ $\vdots$

Algebraic Distance, h_{33}=1 (cont)

Linear equations

$$\underset{2N\times 8}{A}\ \underset{8\times 1}{h} = \underset{2N\times 1}{b}$$

Solve:

$$\underset{8\times 2N}{A^T}\ \underset{2N\times 8}{A}\ \underset{8\times 1}{h} = \underset{8\times 2N}{A^T}\ \underset{2N\times 1}{b}$$

$$\overbrace{(A^T\ A)}^{8\times 8}\ \underset{8\times 1}{h} = \overbrace{(A^T\ b)}^{8\times 1}$$

$$h = (A^T\ A)^{-1}\ (A^T\ b)$$

This is know to be optimal in L.S sense

Problem?

What might be wrong with setting $h_{33} = 1$?

If h_{33} actually $= 0$, we can't get the right answer.

Algebraic Distance, ||h||=1

||h|| = 1

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}}$$

Multiplying through by denominator

$$(h_{31}x + h_{32}y + h_{33})x' = h_{11}x + h_{12}y + h_{13}$$

$$(h_{31}x + h_{32}y + h_{33})y' = h_{21}x + h_{22}y + h_{23}$$

Rearrange

$$h_{11}x + h_{12}y + h_{13} - h_{31}xx' - h_{32}yx' - h_{33}x' = 0$$

$$h_{21}x + h_{22}y + h_{23} - h_{31}xy' - h_{32}yy' - h_{33}y' = 0$$

Algebraic Distance, ||h||=1 (cont)

$$\underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x'_1 & -y_1x'_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y'_1 & -y_1y'_1 & -y'_1 \\ & & & & \vdots & & & & \end{bmatrix}}_{2N \times 9} \underbrace{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}_{9 \times 1} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \end{bmatrix}}_{2N \times 1}$$

2N x 9 — 9 x 1 — 2N x 1

4 POINTS

additional points

Algebraic Distance, ||h||=1 (cont)

Homogeneous equations

$$\underset{2N\times 9}{A}\ \underset{9\times 1}{h} = \underset{2N\times 1}{0}$$

Solve:

$$\underset{9\times 2N}{A^T}\ \underset{2N\times 9}{A}\ \underset{9\times 1}{h} = \underset{9\times 2N}{A^T}\ \underset{2N\times 1}{0}$$

$$\overbrace{(A^T\ A)}^{9\times 9}\ \underset{9\times 1}{h} = \underset{9\times 1}{0}$$

$$\text{SVD of } A^T A = U\ D\ U^T$$

(Singular Value Decomposition)

→ Solution in SVD is proportional to the eigenvector corresponding to the zero eigenvalue of $A^T A$

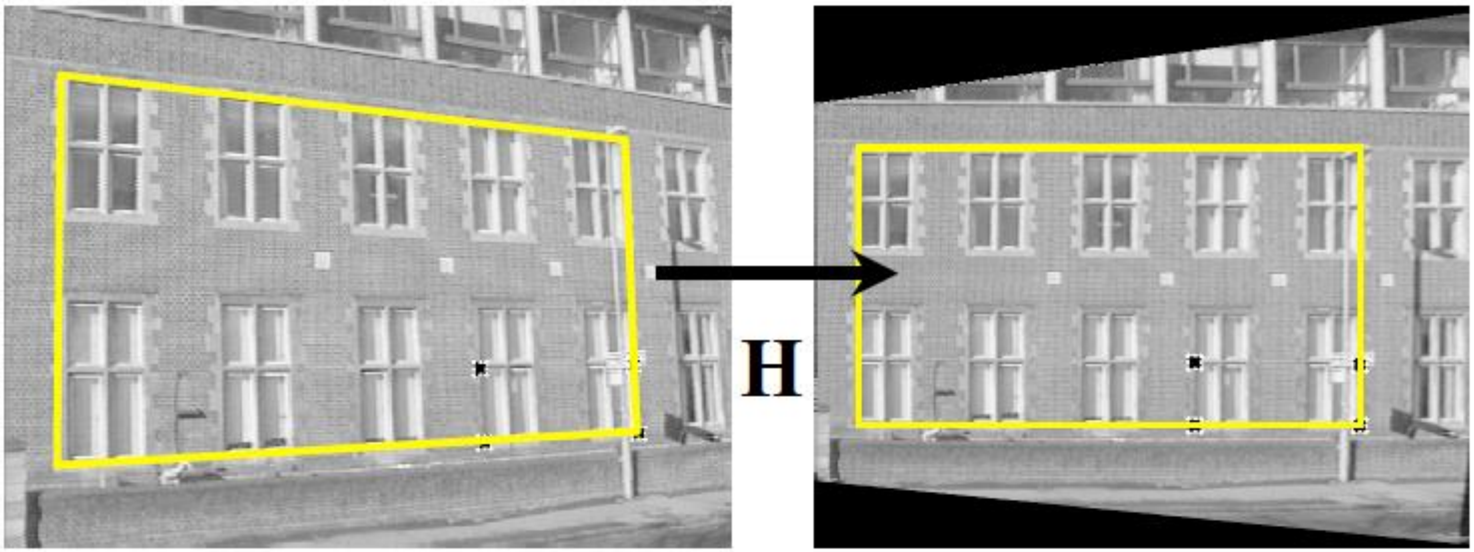
Warping & Bilinear Interpolation

Given a homography between two images, (coordinate systems) we want to "warp" one image into the coordinate system of the other.

We will call the destination coordinate system the "reference" image.

We will call the source coordinate system the "source" image (duh)

Projective Warping Example

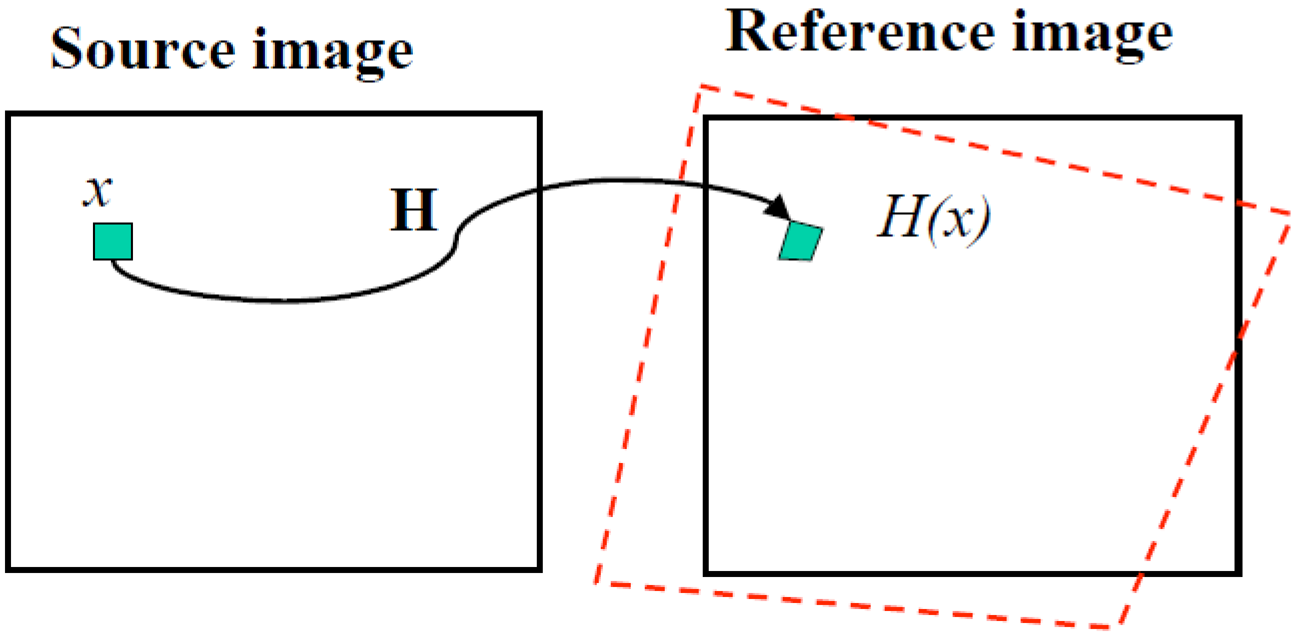


from Hartley & Zisserman

Source Image

Reference image

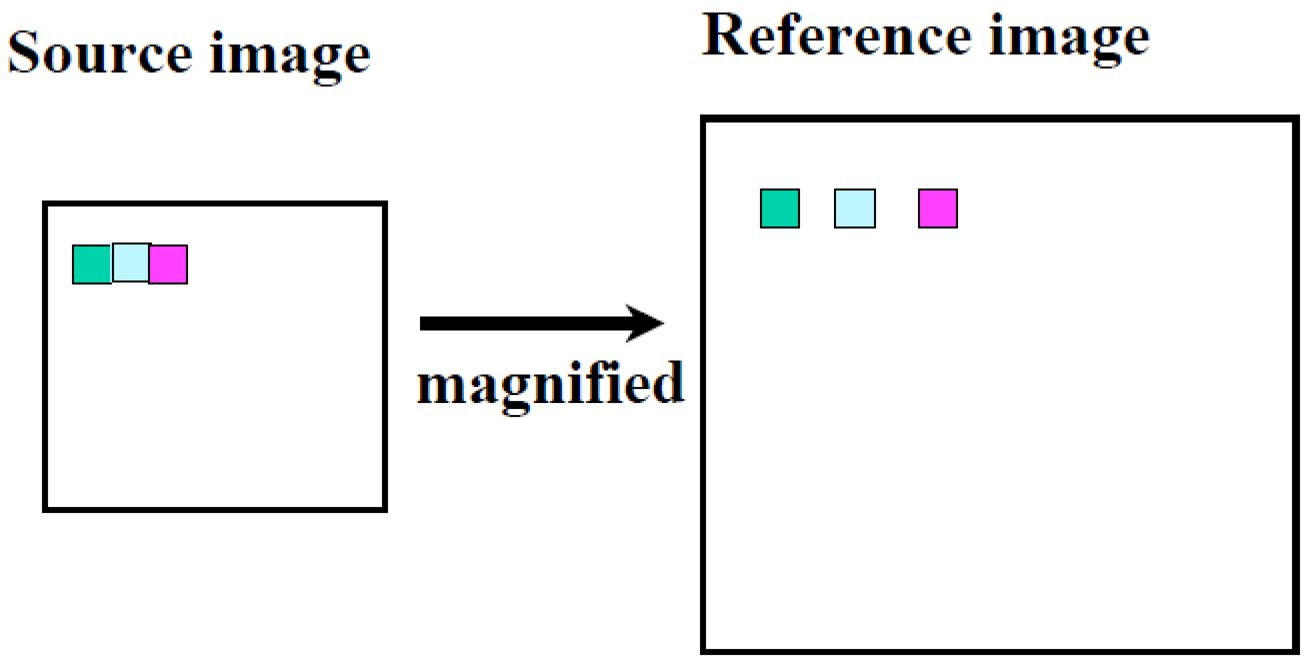
Forward Warping



- For each pixel *x* in the source image
- Determine where it goes as *H(x)*
- Color the destination pixel

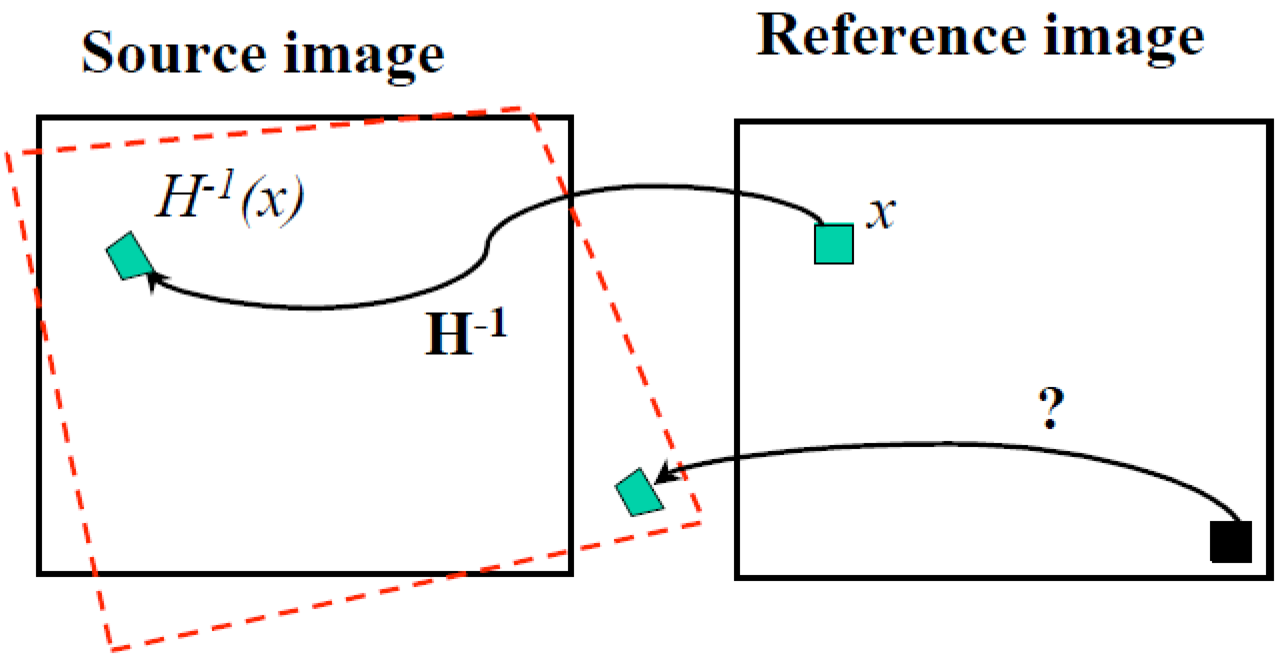
Problems?

Forward Warping Problem



Can leave gaps!

Backward Warping (No gaps)

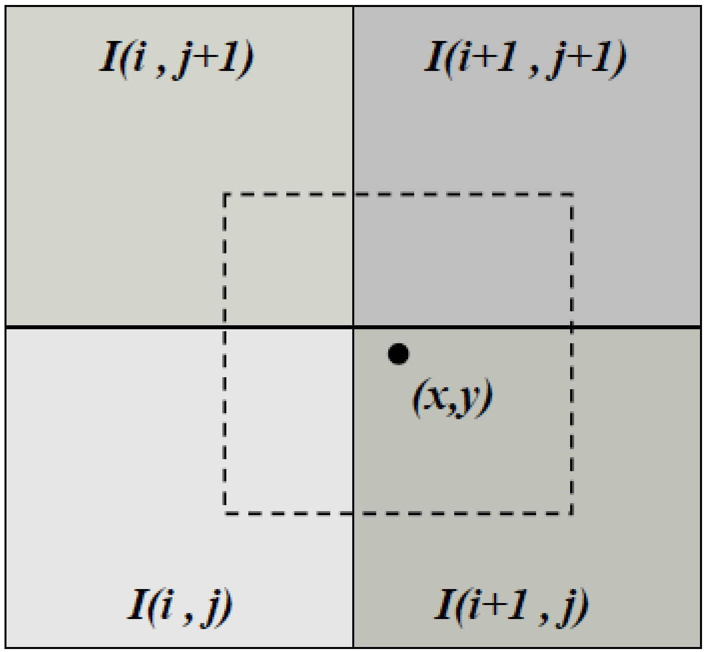


- For each pixel x in the reference image
- Determine where it comes from as $H^{-1}(x)$
- Get color from that location

Interpolation

What do we mean by "get color from that location"?

Consider grey values. What is intensity at (x,y)?



Nearest Neighbor:

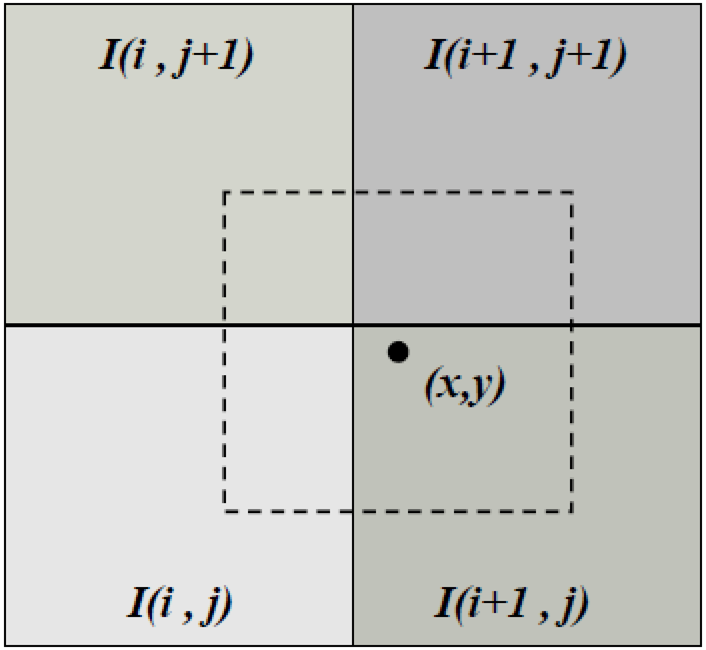
Take color of pixel with closest center.

$$I(x,y) = I(i+1,j)$$

Bilinear interpolation

What do we mean by "get color from that location"?

Consider grey values. What is intensity at (x,y)?



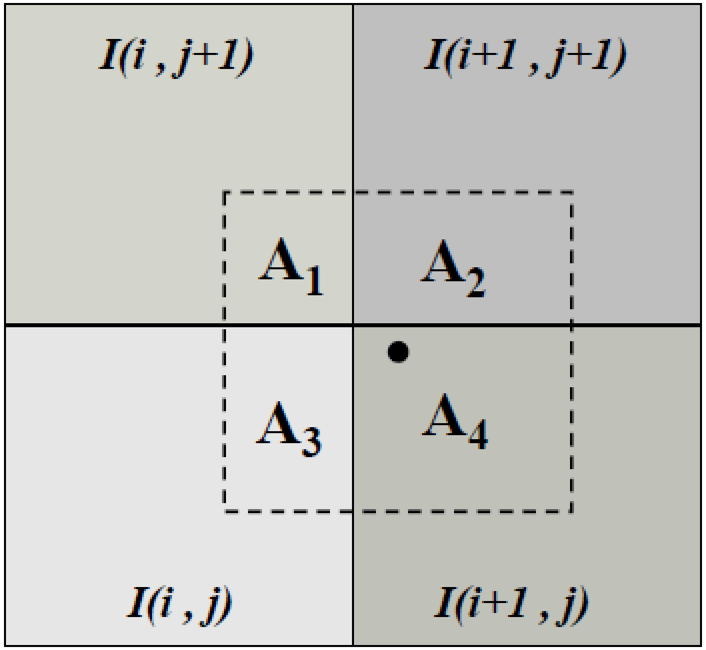
Bilinear Interpolation:

Weighted average

Bilinear interpolation

What do we mean by "get color from that location"?

Consider grey values. What is intensity at (x,y)?

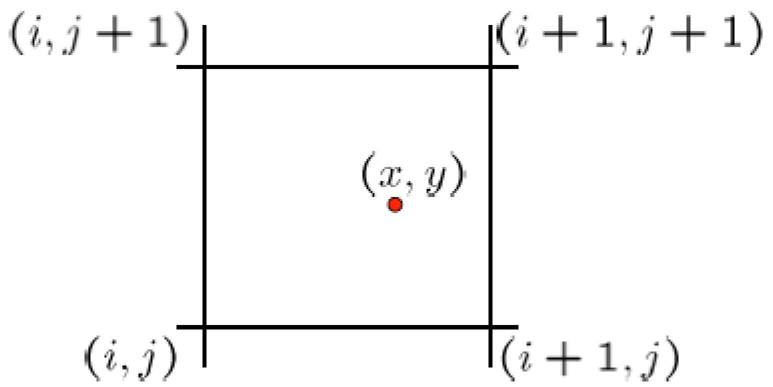


Bilinear Interpolation:

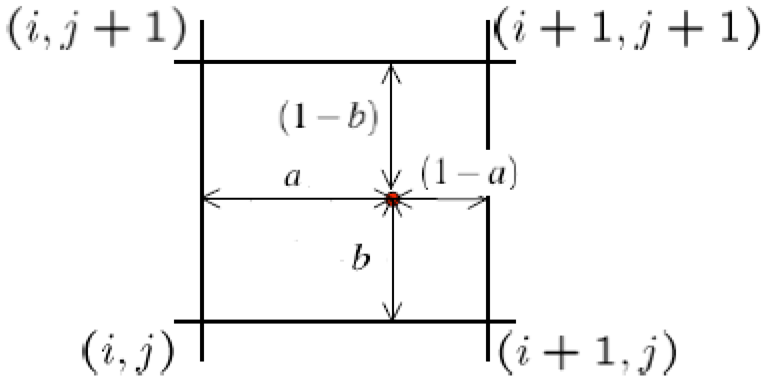
Weighted average

$$I(x,y) = A3*I(i,j) + A4*I(i+1,j) + A2*I(i+1,j+1) + A1*I(i,j+1)$$

Bilinear Interpolation



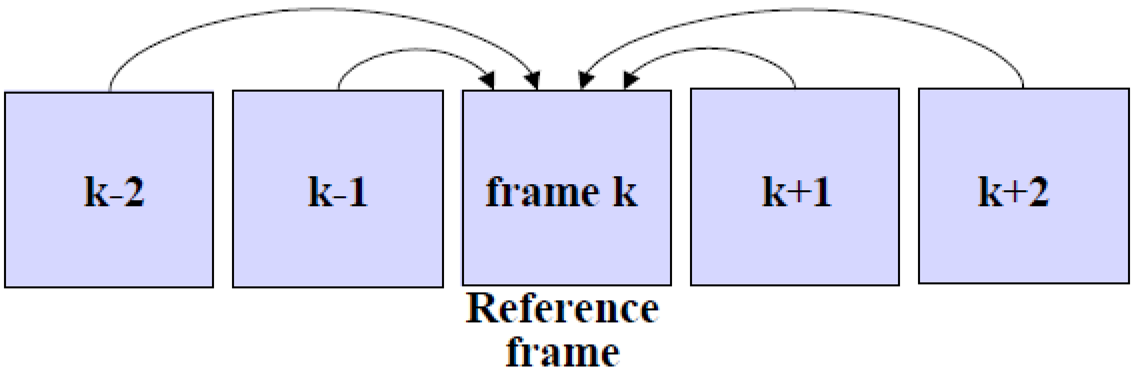
Bilinear Interpolation



$$
\begin{aligned}
\mathbf{I}(x,y) = \; & (1-a)(1-b) && \mathbf{I}(i,j) \\
& +a(1-b) && \mathbf{I}(i+1,j) \\
& +ab && \mathbf{I}(i+1,j+1) \\
& +(1-a)b && \mathbf{I}(i,j+1)
\end{aligned}
$$

Applications: Stabilization

Given a sequence of video frames, warp them into a common image coordinate system.



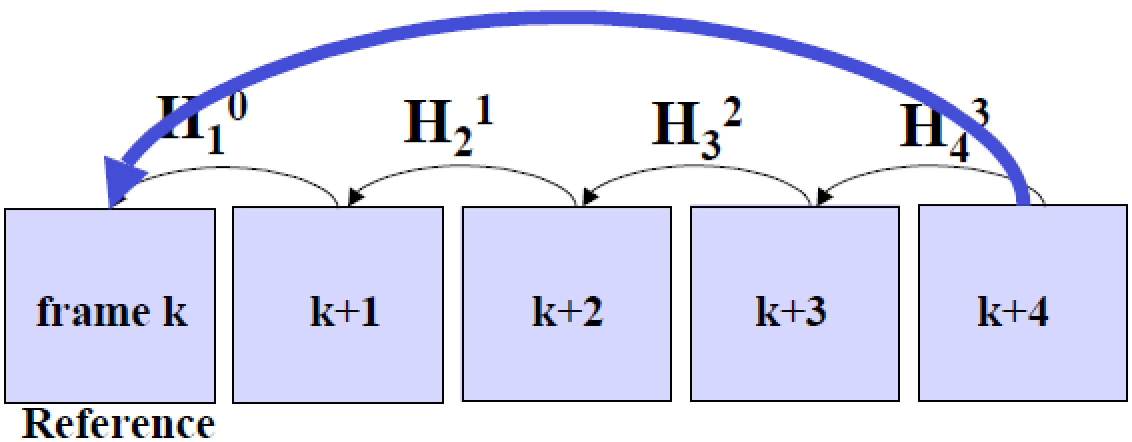
This "stabilizes" the video to appear as if the camera is not moving.

https://www.youtube.com/watch?v=i5keG1Y810U

Stabilization by Chaining

What if the reference image does not overlap with all the source images? As long as there are pairwise overlaps, we can chain (compose) pairwise homographies.

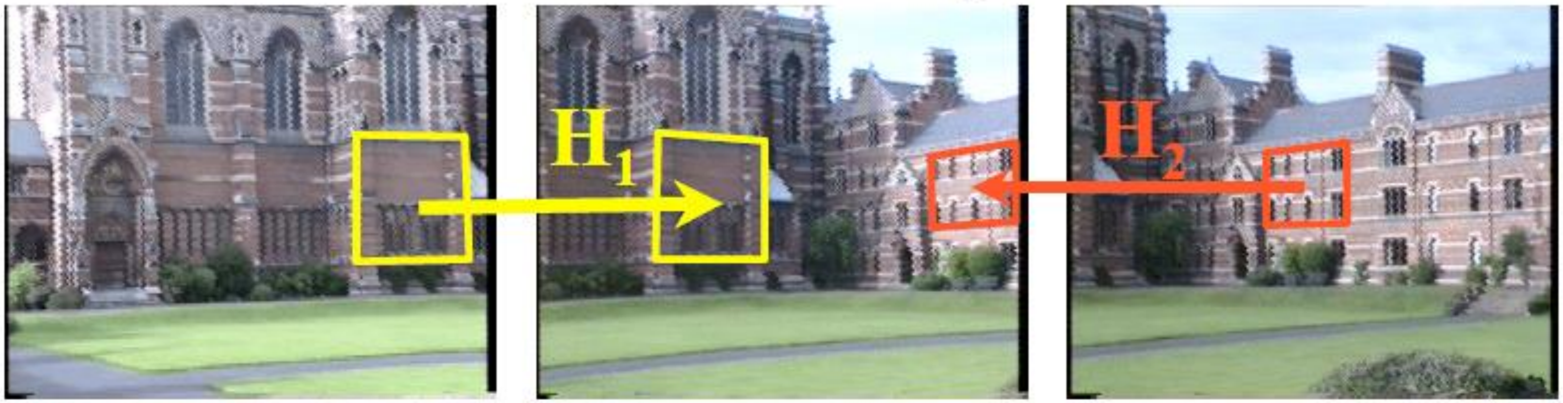
$$H_4^0 = H_1^0 * H_2^1 * H_3^2 * H_4^3$$



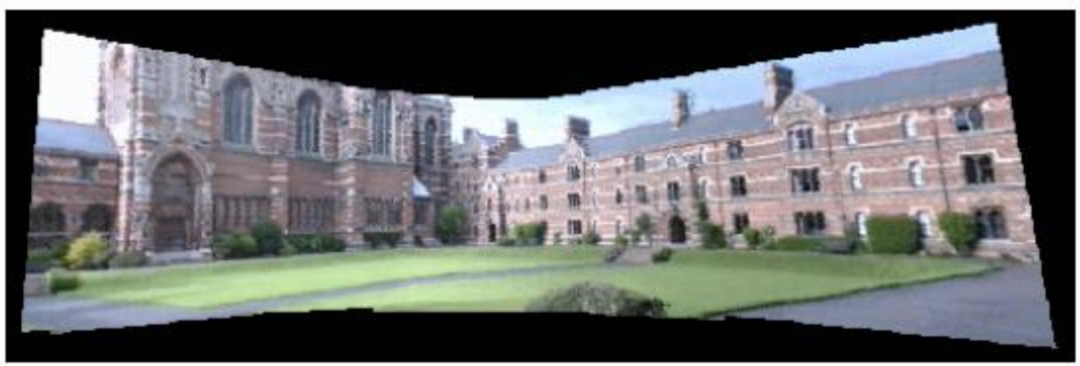
Not recommended for long sequences, as alignment errors accumulate over time.

Applications: Mosaicing

Source 1 **Reference image** **Source 2**



from Hartley & Zisserman



Mosaic

Note on Planar Mosaicing

Assumes scene is roughly planar.

What if scene isn't planar? Alignment will not be good if significant 3D relief

→ "Ghosting"

Ghosting Example

Source image

Reference image

Ghosting Example (cont)

Mosaic

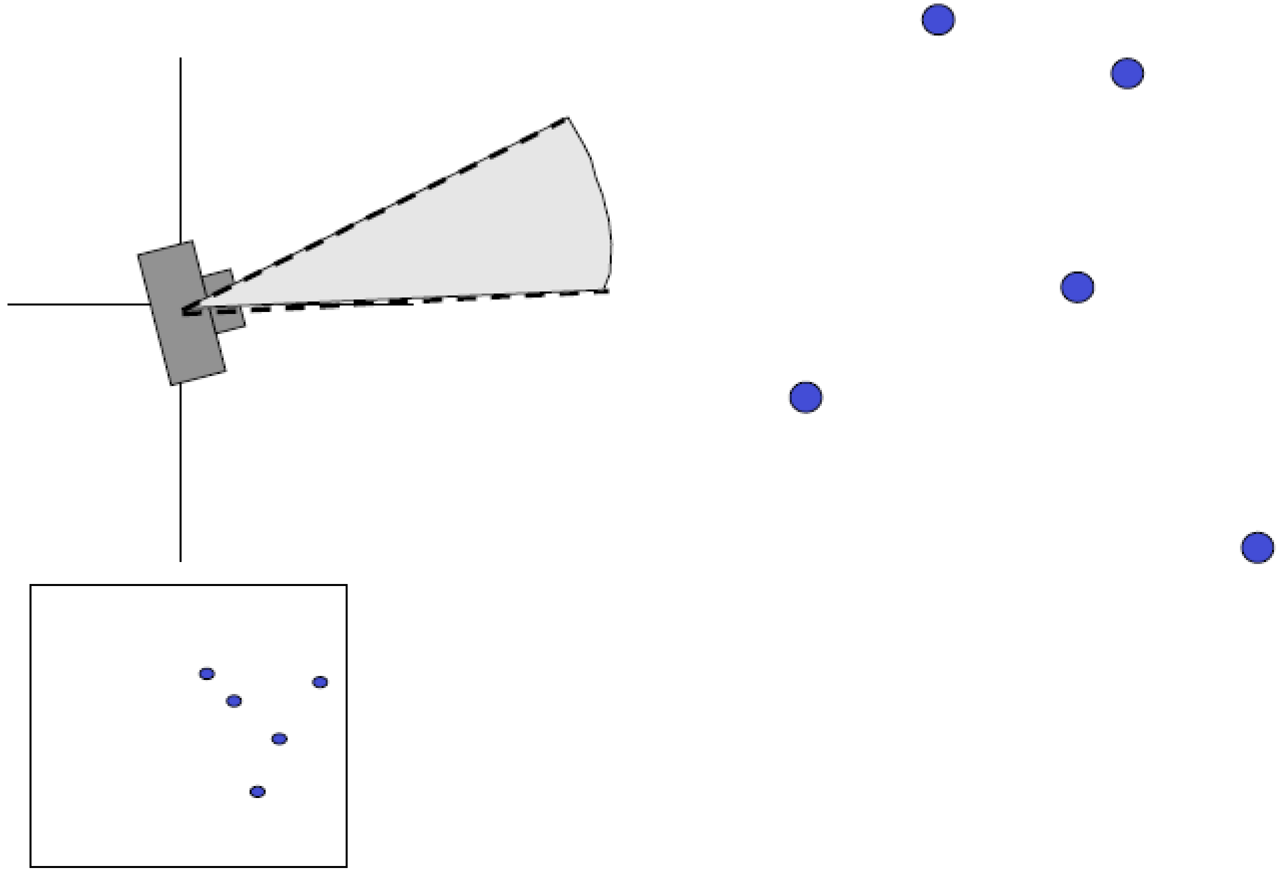
Mosaics from Rotating Cameras

However, there is a mitigating factor in regards to ghosting…

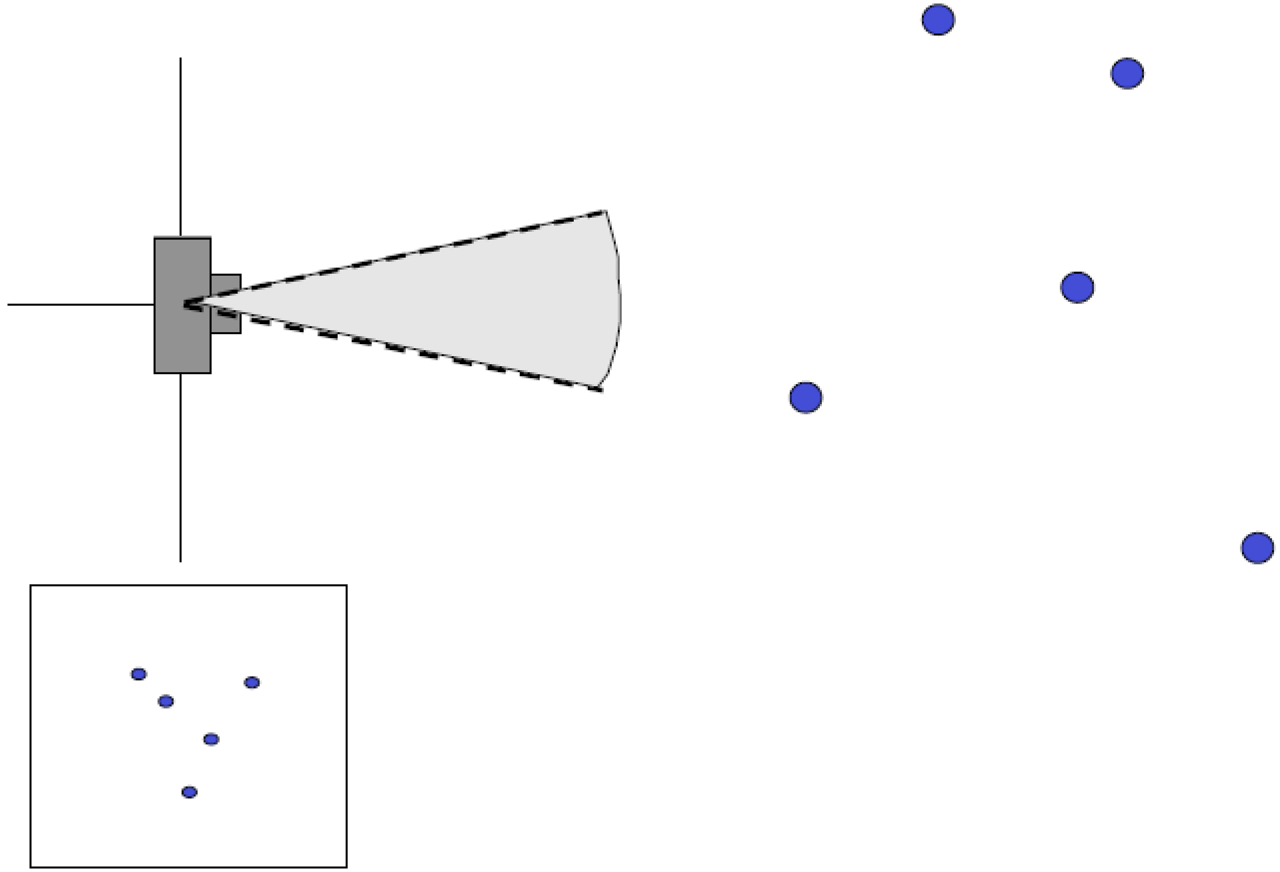
Images taken from a rotating camera are related by a 2D homography…

<u>regardless of scene structure!</u>

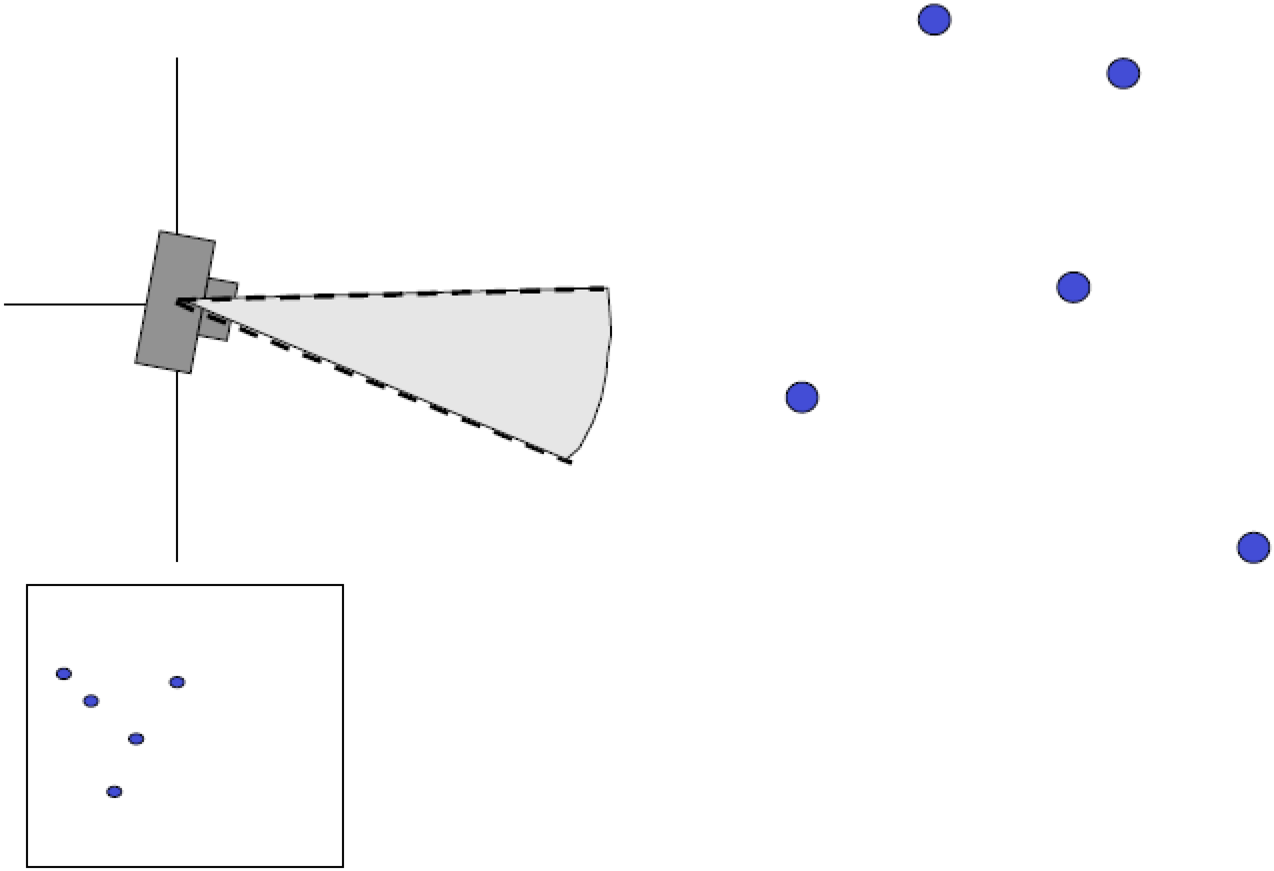
Rotating Camera (top-down view)



Rotating Camera (top-down view)



Rotating Camera (top-down view)



Relations among Images Taken by Rotating Camera

Image 1

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Same ray!

Image 2

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} \sim \begin{bmatrix} h'_{11} & h'_{12} & h'_{13} \\ h'_{21} & h'_{22} & h'_{23} \\ h'_{31} & h'_{32} & h'_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} h'_{11} & h'_{12} & h'_{13} \\ h'_{21} & h'_{22} & h'_{23} \\ h'_{31} & h'_{32} & h'_{33} \end{bmatrix}^{-1} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Mosaicing Example

Original Images (from a pan/tilt camera)

Panoramic (Mosaic) View

One more detail: Blending!

Approaches to Blending

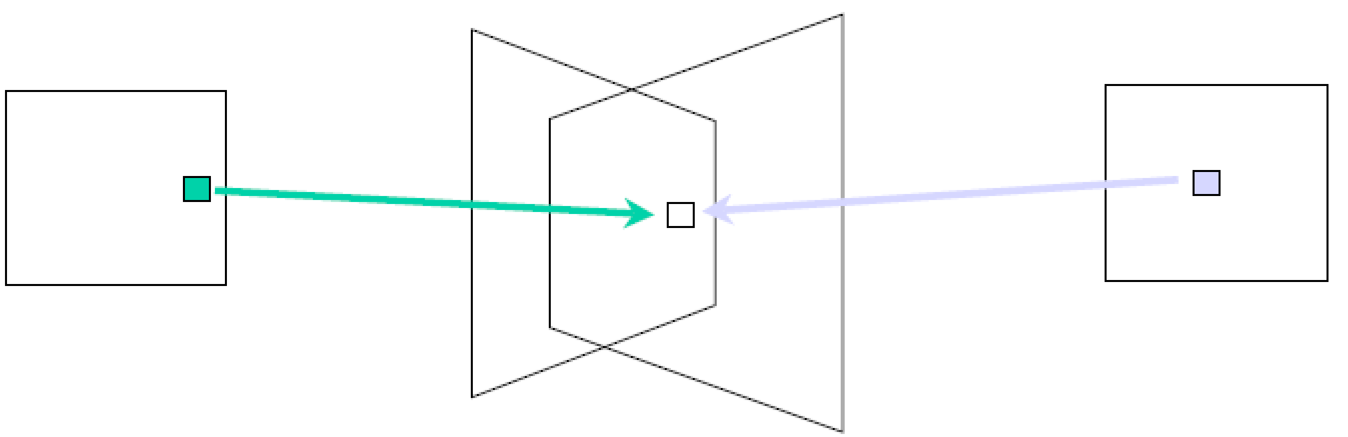
How to combine colors in area of overlap?

1) Straight averaging $\mathbf{P = (P_1 + P_2) / 2}$

2) Feathering $\mathbf{P = (P_1/w_1 + P_2/w_2) / (w_1+w_2)}$

With w_i being distance from image border

3) Equalize intensity statistics (gain, offset)



Before and After Blending